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PROTECTED LIFETIME INCOME BENEFITS: THE NEXT GENERATION GLWB

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ABSTRACT

An increasing number of solutions are being created to help retirees address longevity risk. One example would be a strategy that provides lifetime income where the benefit amount evolves throughout retirement based on the performance of the account, an approach referred to as a "Protected Lifetime Income Benefit" (PLIB). This piece contrasts the efficacy of PLIBs against other common annuity types and notes that they tend to generate the most income for a retiree among the approaches considered, on average, while variable annuities with a guaranteed lifetime withdrawal benefit (GLWB) generated the least. While PLIBs have higher levels of income variability than other annuities, this variability needs to be placed in the correct context, since almost all retirees already have effectively fixed existing guaranteed lifetime income sources, such as Social Security retirement benefits. Overall, this research suggests that PLIB strategies should be considered by financial advisors and retirees when creating efficient retirement income plans for clients.

INTRODUCTION

Annuity sales have increased significantly in the past year as the Baby Boomer population reaches peak retirement age, but even with this sales surge America's retirees are still significantly under-annuitized.

Despite more than a half century of research detailing the potential value of guaranteed lifetime income products for retirees, sales of annuities remain lower than makes sense for a generation of retirees who are leaving private sector work often without benefit of a pension. There are a variety of theories that exist to explain why this "annuity puzzle" persists, but one notable (potential) barrier to annuitization is the irrevocable transfer of the premium common among annuities.

A product that addresses this concern, available since the 1990s, is an annuity with a "Guaranteed Lifetime Withdrawal Benefit" (GLWB) feature,[1] which is common with both variable annuities (VAs) and fixed indexed annuities (FIAs). These products guarantee some minimum level of lifetime income even if the underlying account value goes to zero.

GLWBs have recently fallen out of favor among some insurers, with a growing number of companies exiting the business. In response, new products are being introduced where the guaranteed income amount "evolves" during the payout phase based entirely on returns of the account, a product we refer to as a "Protected Lifetime Income Benefit" (PLIB). While similar to GLWBs, the PLIB is categorized separately because the way the income changes is materially different and the income can decline. The PLIB concept is not new, with tontines[2] being one of the earliest examples of products that provide protected lifetime income with a form of "shared" risk exposure.

1. Also called a "Guaranteed Minimum Withdrawal Benefit" or GMWB
2. Note, this analysis focuses on products that provide an explicit guarantee with respect to the mortality component (i.e., payouts are not affected by deviations in mortality forecasts).

This article explores the efficacy of PLIBs using a utility framework. PLIBs are contrasted against a retirement income strategy that does not include an annuity, as well as strategies which allocate to either a single premium immediate annuity (SPIA), a deferred income annuity (or DIA, which could be a Qualified Longevity Annuity Contract, or QLAC, if purchased in a qualified account assuming certain provisions are met), and a GLWB.

Overall, we find that the potential benefit of the various approaches vary significantly depending on the specific client situation. One of the most important determinants of efficiency is the portfolio withdrawal rate. While the relative difference in the efficacy of the strategies for well-funded retirees (i.e., with lower withdrawal rates) and the potential benefits associated with annuitization (regardless of type) are relatively small,[3] the benefits of annuitization increase for higher withdrawal rates, especially for the PLIB among the strategies considered.

A balanced risk level (e.g., 50%ish stocks) appears to be generally optimal for a PLIB, although the optimal risk level varies significantly for people depending on situation and preferences. The notable variation in optimal risk levels for the PLIB suggests the ability to personalize the risk in a PLIB is likely significantly more important than a GLWB, where higher risk is generally optimal given the nature of the benefit. There may be restrictions on risk levels in certain types of PLIBs, as well as fees that may vary depending on how the portfolio is invested, which will need to be considered when selecting the appropriate product/strategy.

Lower assumed returns reduce the efficacy of the compared to other annuities which provide a more "fixed" benefit. This is not a surprise, since the income from the PLIB is more explicitly linked to portfolio performance and should be considered when selecting both the product itself and the underlying risk level.

Some readers may be surprised by the relative efficiency of the PLIB annuities in the analysis versus the more traditional annuities (i.e., SPIAs and DIAs) considered. There are a number of factors that improve the efficiency of PLIB (and GLWB) strategies such as lapsation, mortality experience, and return expectations, which are reviewed here.

PLIB strategies are likely to be available overlaying a growing number of investment approaches, such as in more traditional portfolios (i.e., a Contingent Deferred Annuity), as well as within an annuity, layered on top of FIAs, VAs, as well as Registered Index-Linked Annuities (RILAs). The potential advantages and disadvantages to each approach are beyond the scope of this piece and will be discussed in future research.

Overall, this research suggests that PLIB strategies are an exciting evolution of GLWBs and should be considered as part of a holistic income strategy for retirees.

GLWBS AND PLIBS: AN OVERVIEW

Early research on the potential benefits of annuitization primarily focused on more traditional, relatively simple guaranteed income products such as single premium immediate annuities (SPIAs), also referred to as immediate fixed annuities. Despite the noted potential benefits of SPIAs, and annuities in general, they remain relatively unpopular among retirees today, an effect commonly dubbed the "annuity puzzle."

While there are a variety of potential explanations for the annuity puzzle, one notable barrier is irrevocable design of many annuities (e.g., SPIAs), which require the annuitant to permanently cede the premium to receive the lifetime income. The irrevocable aspect of the product is likely the primary reason sales of SPIAs and DIAs are relatively low compared to other income strategies.

One product that provides lifetime income yet also allows on-going access to the premium would be a strategy with a "Guaranteed Lifetime Withdrawal Benefit" (GLWB) feature, also sometimes referred to as a "Guaranteed Minimum Withdrawal Benefit" or GMWB. These are common in both variable annuities (VAs) and fixed indexed annuities (FIAs) and could be offered with more traditional portfolios as part of a Contingent Deferred Annuity (CDA).

3. This is not surprising, since an annuity represents a form of insurance for portfolio ruin, if the withdrawal amount is relatively small the probability of ruin is low reducing the value of the insurance.

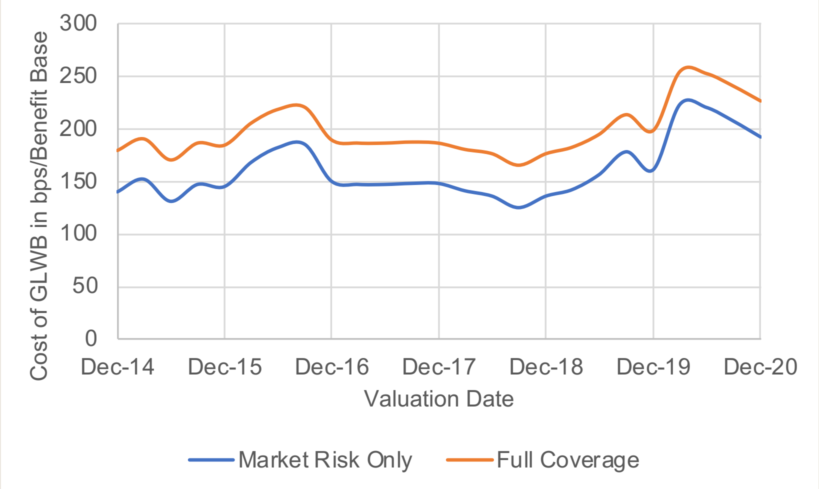

EXHIBIT 1. *The Cost of Reinsuring a GLWB*

Source: *Munich RE (2021)*

Introduced in the 1990s, GLWBs allow access to the contract value (i.e., are revocable) and guarantee some minimum level of lifetime income (which could potentially increase) even if the underlying account value goes to zero.

GLWBs have somewhat gone out of favor recently, with a number of providers exiting the business, given the low bond yield environment and the rising costs associated with issuing them. For example, Munich RE (2021) has released research estimating how the reinsurance cost of GLWBs has changed from January 2000 to December 2020, which is included in Exhibit 1.

Munich RE has two potential cost estimates[4] and the cost of either has increased significantly, recently exceeding 200 bps, well above current fee levels, which tend to be 150 bps less.

In response to the challenging economics associated with issuing GLWBs there has been an increasing number of products that offer lifetime income, but where the income amount "evolves" based on the returns of the underlying portfolio, for better or worse, without the same income floor as GLWBs (i.e., the income level can decline). We refer to these products as "Protected Lifetime Income Strategies" (PLIBs). While PLIBs are relatively similar to GLWBs, and could theoretically fall under the same general lifetime income umbrella, a separate naming approach is used given the notable difference in how the income can change during retirement.

BACK TO THE FUTURE TO SOLVE THE ANNUITY PUZZLE

The PLIB structure is not new. For example, one of the oldest longevity products that could fall under the PLIB umbrella would be a tontine, which is an annuity structure devised in the 17th century where annuitants (also called subscribers, shareholders, investors, etc., depending on the structure) share in the investment and mortality experience of the pool. While tontines are not widely available today, there is growing interest in the structure and some recent products designed around the approach.

4. The "Market Risk Only" level of the RCI uses a reinsurance structure that provides similar risk protection to a complete market-risk hedging program covering all relevant greeks, while also reinsuring all cross-greeks and operational risks associated with a hedging program. The "Full Coverage" level of the RCI uses a reinsurance structure that transfers all material risks, including non-hedgeable risks such as behavior risk and basis risk (with the exception of post-claim longevity risk which is not transferred because the reinsurance claim is paid as a lump sum).

This analysis assumes that only the investment returns affect the PLIB benefits, not the mortality experience, which implies the backing of an insurance company. While tontines could theoretically be included in the analysis, miscalculations regarding mortality experience can have significant implications for the annuitants who survive the longest. By focusing on an "insured" PLIB structure we can more easily contrast the potential benefits of a strategy with other strategies that have a different payout approach (e.g., GLWBs) but which also explicit have mortality guarantees.

PAYOUT MECHANICS

This section provides an overview of the payout mechanics of GLWBs and PLIBs.

GLWB MECHANICS

The income level in a GLWB is determined based on applying the payout rate, also known as the guaranteed percentage or lifetime distribution factor, to the "benefit base" (also called the income base). The payout rate is based on the age of the annuitant at the time of the first withdrawal, or the younger of the two annuitants if it's a joint couple. GLWB payout rates typically increase at older ages at varying increments (e.g., some companies have five-year bands, other more granular levels) and are typically higher for single versus joint annuitants.

The benefit base is a type of "shadow account" (i.e., it is not accessible) used to determine the income level. The benefit base is typically based on the contract policy value of each succeeding anniversary date (i.e., the highwater mark). Some GLWB products have additional valuation methods, such as guaranteed crediting rates, which guarantee minimum increases in the benefit base through time.

For example, if a male retiree, age 65, invested $100,000 in a GLWB with a 5% payout rate, he would be guaranteed at least $5,000 per year for life ($100,000 * 5 percent = $5,000), regardless of the underlying contract value (i.e., if it goes to zero). If the annuity portfolio value were to increase to $110,000 (on an anniversary date) the benefit base would "step up" to $110,000 and the guaranteed lifetime income amount would increase to $5,500 ($110,000 * 5 percent = $5,500) and not go below that level for life, regardless of future performance. The benefit base, and corresponding income level, could increase again if the portfolio value reached a new high on a future anniversary date (although this becomes increasingly unlikely over time).

GLWB fees and provisions vary by provider. Since the GLWB rider is essentially a lifetime put option, if the fee associated with the GLWB rider didn't vary by equity allocation, investors would be best served investing in the most aggressive portfolio possible inside the annuity. While it was common to offer a variety of investment options before the market crash of 2008 (i.e., the global financial crisis), the vast majority of insurers today still offering GLWBs require annuitants to select among a few relatively well diversified portfolios or only offer a single investment option once the income begins (e.g., a 60% equity portfolio).

When assessing the potential value of a GLWB, or really an annuity, it's important to focus on the product in its entirety versus a single attribute. For example, just because a given GLWB has a relatively high fee does not necessarily mean the product is lower quality. The higher fee could be offsetting some other benefit that actually makes it relatively more attractive (e.g., a higher payout rate). This can make accessing the relative value of a GLWB difficult given the myriad features associated with the various strategies.

PLIB MECHANICS

Payouts from PLIBs are similar to GLWBs, in that PLIBs provide some amount of guaranteed income for life regardless of the underlying account value (i.e., even if it goes to zero). The key difference is that the guaranteed income from the PLIB changes based on the performance of the account, while the income from a GLWB is based on adjustments to the benefit base. In order for the income from a GLWB to increase in the distribution phase, the return must exceed the distribution amount and the fees, something that becomes increasingly unlikely over time.

The growth in the income from a PLIB is typically going to be based on the credited return minus any applicable fees (i.e., net performance), although gross performance could also be used. For example, if the current

GLWB						
Year#	**Gross Return**	**Net Return**	**Begin Contract Value**	**Begin Benefit Base**	**Income**	**End Contract Value**
1	10.0%	8.5%	$100,000	$100,000	$4,500	$103,618
2	0.0%	-1.5%	$103,618	$103,618	$4,663	$97,470
3	10.0%	8.5%	$97,470	$103,618	$4,663	$100,696
4	0.0%	-1.5%	$100,696	$103,618	$4,663	$94,593
5	10.0%	8.5%	$94,593	$103,618	$4,663	$97,574
6	0.0%	-1.5%	$97,574	$103,618	$4,663	$91,518
7	10.0%	8.5%	$91,518	$103,618	$4,663	$94,238
8	0.0%	-1.5%	$94,238	$103,618	$4,663	$88,231
9	10.0%	8.5%	$88,231	$103,618	$4,663	$90,672
10	0.0%	-1.5%	$90,672	$103,618	$4,663	$84,719

PLIB						
Year#	**Gross Return**	**Net Return**	**Begin Contract Value**	**Begin Benefit Base**	**Income**	**End Contract Value**
1	10.0%	8.5%	$100,000	n/a	$4,000	$104,160
2	0.0%	-1.5%	$104,160	8.50%	$4,340	$98,323
3	10.0%	8.5%	$98,323	-1.50%	$4,275	$102,042
4	0.0%	-1.5%	$102,042	8.50%	$4,638	$95,943
5	10.0%	8.5%	$95,943	-1.50%	$4,569	$99,141
6	0.0%	-1.5%	$99,141	8.50%	$4,957	$92,771
7	10.0%	8.5%	$92,771	-1.50%	$4,883	$95,359
8	0.0%	-1.5%	$95,359	8.50%	$5,298	$88,710
9	10.0%	8.5%	$88,710	-1.50%	$5,218	$90,589
10	0.0%	-1.5%	$90,589	8.50%	$5,662	$83,653

EXHIBIT 2. *Income Example for a GLWB and PLIB*

income from a PLIB was $5,000 and the net return of the account (including fees) for the prior period was +20% the income level would increase by 20% to $6,000. In this case the income would increase regardless of the underlying value of the contract.

Since the PLIB payout is solely focused on returns, the probability of an income increase is significantly higher with a PLIB versus a GLWB; however, the reverse is also true, whereby the income from a PLIB can decline if the returns are negative (while they would not for a GLWB). For example, if net return of the PLIB account was -20%, an income of $5,000 would drop to $4,000. The income could drop even further depending on account performance.

The income level from a PLIB will typically remain constant (for the life of the annuitant) once the account has been depleted and be based on the previous year's value (before the account is exhausted).

PANEL A: GLWB INCOME DISTRIBUTION   PANEL B: PLIB INCOME DISTRIBUTION (40% EQUITY)

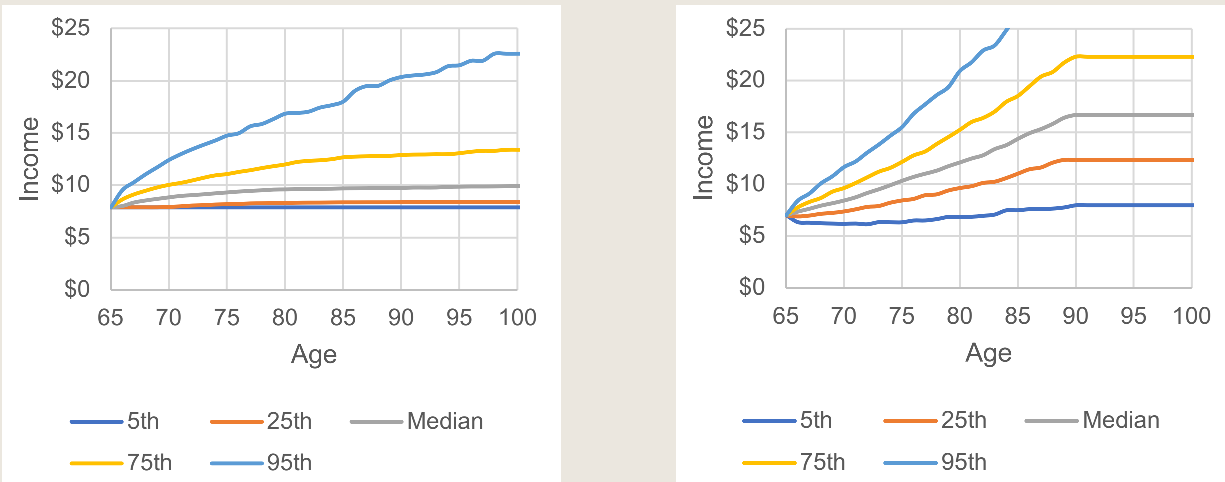

EXHIBIT 3. *Distribution of Income Throughout Retirement*

COMPARISON

Exhibit 2 provides an example of how the income would change over time for a GLWB and PLIB assuming the same initial contract value ($100,000) and returns. The gross returns are assumed to alternate between 10% and 0% each year and the fee is 1.5%, which is assumed to be assessed against the contract value (effectively reducing the credited return). The initial payout for the GLWB is assumed to be 4.5% versus 4.0% for the PLIB, which reflects existing market payout dynamics for a 65-year-old couple (where initial GLWB payout rates tend to be higher than PLIBs).

The initial income level is higher for the GLWB ($4,500 versus $4,000); however, by the end of the 10-year test period the PLIB income level is higher ($5,662 versus $4,663). This can primarily be attributed to the positive average return over the period (5% gross and 3.5% net). While the income from the GLWB increases in the first year, there are no subsequent increases because the contract value never again reaches a new highwater mark. This is in contrast to PLIB, where the income level changes annually based on realized returns, increasing in years with the positive net returns and decreasing in years with the negative net returns. By freeing the income growth level from the benefit base, the PLIB provides significantly more upside with respect to income, but also subjects the annuitant to more downside.

The potential distribution of the income from a GLWB and PLIB is demonstrated in Exhibit 3, which is based off the key assumptions outlined in the following section. The GLWB is assumed to have a 60% equity allocation (the maximum possible) while the PLIB is assumed to have a 40% equity allocation (more conservative to balance the risk associated with portfolio declines and the subsequent reduction in lifetime income).

The income distribution for the GLWB is clearly tighter than for the PLIB. While there are a few scenarios where the income level increases considerably for the GLWB, the increases tend to be relatively small, and occur at a decreasing rate throughout retirement. In contrast, the income levels from the PLIB increasingly diverge over time reflecting the cumulative returns (and volatility) of the portfolio over time. Positive returns generally benefit the PLIB more than the GLWB, since the income level for the PLIB is based entirely on the credited return (versus achieving a new highwater mark for the GLWB). Again, the PLIB income is assumed to remain constant after the portfolio value is exhausted, which is why the income distribution flattens for the PLIB at older ages.

THE OPTIMAL STRATEGY

In this section a model is introduced to determine the optimal retirement income funding strategy given a variety of scenarios and possibilities. The optimal retirement income strategy is determined using an approach based off the Constant Relative Risk Aversion (CRRA) utility function, shown in equation 1, where the amount of utility (U) received varies depending on level of consumption (c) and level of investor risk aversion (γ).

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad [1]$$

A utility-based approach is used, versus other more commonly used metrics among financial advisors such as the probability of success, since it can better capture the economic implications of shortfall in retirement. Implied within the CRRA utility function is the law of diminishing marginal utility, whereby negative outcomes (especially extreme negative outcomes) are weighted more heavily than positive outcomes. The specific utility approach used in this research is a modified version of the approach introduced by Blanchett and Kaplan (2013) and is described in detail in Appendix 1 and overviewed more in the following section.

The analysis assumes the household is a male and female couple, both age 65, retiring immediately with $500,000 in savings. Note, the savings amount assumed for the analysis is not that important, rather it is the saving relative to other sources of retirement income assumed.

Taxes are effectively ignored for the analysis; however, there can be different taxation structures for different types of annuities, and therefore some of the findings could potentially change if taxes were fully considered as part of the analysis.[5]

While we are primarily interested in comparing the efficacy of GLWBs and PLIBs, since they are most similar in structure, other annuities are included for robustness purposes. In other words, just because a PLIB is better than a GLWB (for example) does not necessarily mean it's better than other potential strategies, and it should be placed in the larger context of other annuities available to retirees.

Four different types of annuities are considered: an immediate annuity (SPIA), a deferred income annuity (DIA), a GLWB, and a PLIB. The allocations by product vary to result in approximately the same level of income for the products which generate income immediately (SPIA, GLWB and PLIB) while the DIA generates roughly double the income of the immediate-income products.

The payout rate for the SPIA is determined by averaging the five highest available quotes obtained from CANNEX on April 27, 2022 for a 65-year old couple, male and female, with a 100% continuation benefit and a 20-year period certain rider. The average payout was 5.45%. While research commonly assumes retirees purchase life-only annuities, only a minority of annuities quoted are life-only (and likely even a lower percentage of sales). For example, only 12.87% of the 669,574 annuities quoted by CANNEX (2021) in the calendar year 2020 were life-only. The SPIA allocation is assumed to be 30% of the initial balance.

The payout rate for the DIA is determined by averaging the five highest available quotes obtained from CANNEX on April 27, 2022 for a 65-year old couple, male and female, with a 100% continuation benefit with a cash refund provision where payments commence at age 80. The average payout was 14.92%. The DIA allocation is assumed to be 20% of the initial balance.

The GLWB is assumed to have a 4.5% payout rate[6] with a 60% equity allocation and 1.5% total annual fee, which is assessed against the contract value. This is consistent with institutionally priced products today, especially those available in the fee-only advisor space or in defined contribution plans. The GLWB allocation is assumed to be 35% of the initial balance and the benefit base is assumed to potentially step-up annually if the contract value exceeds the previous year's benefit base.

The PLIB is assumed to have a 4.0% initial payout rate with a 1.5% total annual fee. Income from the PLIB changes based on account performance, which is reduced by the total contract fee (i.e., 1.5%). In other words, if the PLIB achieves a return of 0% per year, the

5. For example, SPIAs are taxed using an exclusion ratio approach, while all gains are usually taxed first for GLWBs, although some GLWB products also use a tax exclusion approach.
6. While 5% is a common GLWB payout at age 65, as a reminder, this analysis is for a joint couple, which is why the payout is lower (4.5%).

income amount would decline by 1.5% (the fee). The base assumed equity allocation for the PLIB is 40%, to reflect the higher risk implications of lower returns (i.e., the subsequent reductions in lifetime income), although the equity allocation is varied as part of the analysis to determine optimal equity levels across scenarios. The income from the PLIB is assumed to remain constant once the account value is depleted. The PLIB allocation is assumed to be 35% of the initial balance.

Opposed to testing a single set of household attributes, a variety of scenarios are considered by varying four key parameters:

1. Portfolio Equity Allocation: this is the equity allocation for the investment portfolio (i.e., the monies that are not annuitized) and is assumed to remain constant for the duration of retirement. The portfolio equity allocation is assumed to be exogenous from the annuity allocation, whereby the portfolio risk level does not change based on the amount or type of annuity allocation. In reality, the annuity allocation should likely impact the risk of the portfolio (e.g., if a client purchases an immediate annuity the portfolio could be invested more aggressively) but it's not clear whether this actually occurs in practice. The low, mid, and high equity allocations tested are 10%, 40%, and 70%, respectively. The total assumed expense ratio of the portfolio is assumed to be .50%.

2. Social Security Retirement Benefits: the amount of existing guaranteed income has a significant impact on the potential benefits of annuitization (i.e., buying more guaranteed income) and it is going to vary significantly by household. For the analysis the household is assumed to have Social Security retirement benefits of $10,000, $30,000, or $50,000. Note, the absolute value of the Social Security retirement benefits is not necessarily important, rather it's the relative amount of the guaranteed income to total savings (which is held constant at $500,000).

3. Shortfall Risk Aversion: this variable captures how an income shortfall would affect a retiree household based on the utility model fully detailed in Appendix 1. Three different risk aversion values are tested: 1, 4, and 16, which are assumed to correspond to very low, mid, and very high risk aversion levels, respectively.

4. Initial Portfolio Withdrawal Rate: opposed to assuming the retiree household follows an "optimal" withdrawal strategy, a variety of initial withdrawal rates are tested to determine how the strategies vary across different funding levels. Households with lower withdrawal rates would be considered better funded for retirement, since the required withdrawal rate will be lower.

The analysis assumed to either be constant for duration of retirement (nominal) or increase annually with inflation (real). While historical research on retiree spending has primarily assumed that portfolio withdrawal (and retiree spending) increases annually with inflation (i.e., a real income level), retiree spending does not tend to increase fully by inflation, and actually declines in real terms (i.e., today's dollars), on average, an effect documented by Blanchett (2014), among others.

Note, the analysis does include Social Security benefits, which increase by inflation (inflation is assumed to be stochastic in the analysis), and those increases are assumed to be consumed by the household. Therefore, the nominal income level applies only to the portfolio withdrawal since part of the retiree spending is assumed to increase annually with inflation (whatever the change in Social Security benefits is assumed to be).

Three separate types of returns are generated for the analysis: inflation, bonds, and stocks. Annual returns for the three asset classes are assumed to be 2.5%, 3.5%, and 8.5%, respectively, with standard deviations of 1.5%, 7.0%, and 18.0%, respectively. Returns are assumed to be normally distributed. While the actual historical annual returns of these assets have not been perfectly normally distributed, they have been approximately so, especially at the frequency considered (annual). The correlation between these asset classes is assumed to be zero, which is also roughly consistent with historical values.

The return assumptions for the analysis reflect the current bond yield environment, since the current rate environment plays an important role in annuity pricing (e.g., especially for SPIAs and DIAs). While it's certainly possible bond yields (and the respective payouts for annuities) could increase in the future and return closer to long-term averages, assuming interest rates would rise in the forecast would bias the results (in particular against SPIAs and DIAs, since they are priced based on the current rate environment).

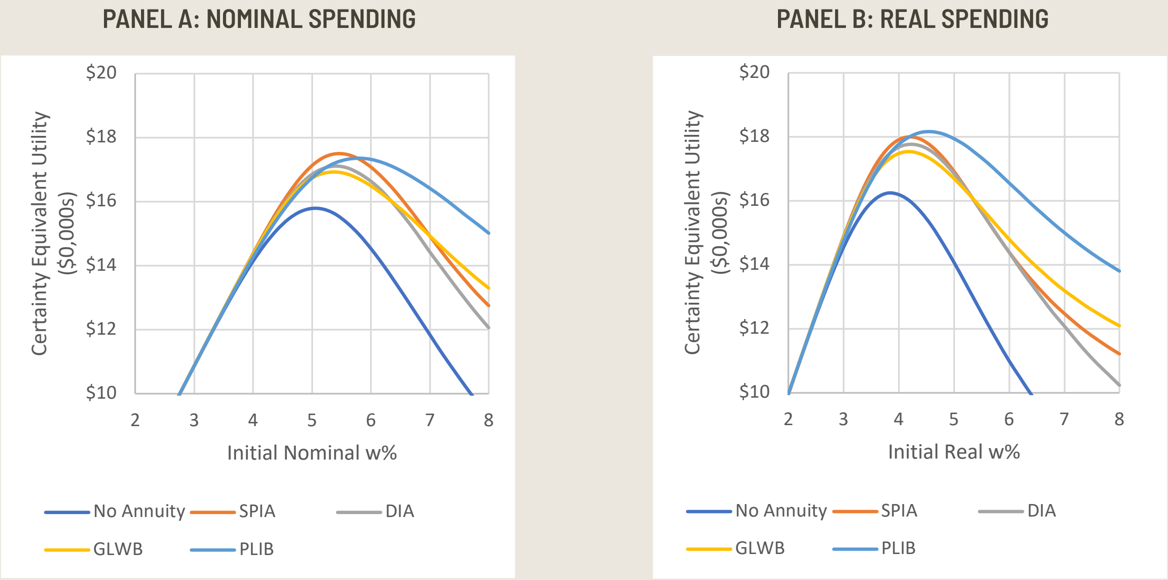

EXHIBIT 4. *Utility of Various Initial Withdrawal Rates Across Strategies*

Mortality rates for the analysis are based on the Society of Actuaries Individual Annuity Mortality (2012 IAM) Table with improvement to year 2022. Mortality rates for the couple are assumed to be independent and the retirement goal is assumed to be the same whether either or both members of the couple are alive.

USING THE UTILITY MODEL TO SELECT THE OPTIMAL STRATEGY

The optimal strategy is determined using a utility model that was overviewed in the previous section and is described more fully in Appendix 1. Utility models are incredibly common in academic literature, but they are rarely used (or available) in financial planning programs, especially those commonly used by financial advisors.

Utility models are generally used to quantify satisfaction, where the higher the resulting utility, the "better" the respective strategy would be deemed to be. When selecting from a variety of potential options the one with the highest utility would be considered optimal. A utility model can be used to compare the efficacy of different retirement income strategies, providing insights into not only whether a household should annuitize, but if so, what type of product would be optimal.

Exhibit 4 includes the utility values for nominal (Panel A) and real (Panel B) initial withdrawal rates from 2% to 8% for the five strategies included in the analysis. The results are based off the moderate base set of assumptions, where the portfolio equity allocation equals 40%, Social Security retirement benefits equal $30,000, and the risk aversion coefficient is 4.

The utility levels are virtually identical at lower initial withdrawal rates (e.g., up to a ~4% nominal withdrawal rate and ~3% real withdrawal rate). This is because the retiree household is unlikely to deplete savings when spending rates from assets are relatively low, which means the specific strategy selected should not materially affect the outcome. This suggests, for example, that retirees with relatively low portfolio withdrawal rates do not necessarily need to annuitize. This is consistent with expectations, since annuities are effectively a form lifetime income insurance, guaranteeing some minimum level of spending, which is going to be of less of a concern if the household has high relative wealth.[7]

7. This focused on the potential economic benefits of annuities and ignores the potential behavioral benefits associated with annuitization, such as higher spending levels.

Initial Nominal Withdrawal Rate (%)						
Strategy	**3**	**4**	**5**	**6**	**7**	**8**
No Annuity	10.87	14.15	15.79	14.54	11.85	9.30
SPIA	10.88	14.38	17.12	17.08	14.93	12.76
DIA	10.88	14.36	16.85	16.63	14.43	12.07
GLWB	10.88	14.36	16.74	16.49	14.93	13.30
PLIB	10.88	14.28	16.74	17.32	16.42	15.02
Initial Real Withdrawal Rate (%)						
Strategy	**3**	**4**	**5**	**6**	**7**	**8**
No Annuity	14.57	16.19	14.08	11.00	8.64	7.04
SPIA	14.91	17.92	16.92	14.40	12.49	11.22
DIA	14.88	17.68	16.85	14.39	12.09	10.24
GLWB	14.85	17.49	16.70	14.80	13.21	12.10
PLIB	14.77	17.77	17.94	16.56	15.01	13.81

Best Strategy Across Withdrawal Rates (shaded) — Best Withdrawal Rate for Strategy (boxed)

EXHIBIT 5. *Utility of Various Initial Withdrawal Rates Across Strategies*

The higher the initial withdrawal rate the higher probability of portfolio depletion and therefore the more likely the household would benefit from an annuity, and the differences in utility across annuities increase as withdrawal rates increase. Other factors also impact the resulting utility estimates, such as the existing guaranteed income sources (i.e., Social Security retirement benefits), income risk aversion levels, etc., which we will discuss more next.

Certain strategies in Exhibit 4 clearly result in more utility (i.e. are better) than others, a point which becomes clearer at higher withdrawal rates (e.g., a 7% withdrawal rate). If we have to rank the strategies based on their utility (from high to low) the results would generally be: PLIB, SPIA, DIA, GLWB, and no annuity. In other words, the PLIB strategy would be considered best among the five, while the no annuity strategy would be considered the worst.

In Exhibit 5 we provide some numerical context around how the model can be used to determine the optimal strategy across the five approaches for each initial withdrawal rate.

If we focus on the real withdrawal rate results in Exhibit 5, if a household was not interested in purchasing an annuity, but wanted to select the utility-maximizing initial real withdrawal rate, that would be 4% (among those included) with a utility value of 16.19 (consistent with the "4% Rule"). Alternatively, if the retiree household wanted a 7% initial nominal withdrawal rate and was interested in determining which of the five potential strategies to implement, the PLIB would be optimal strategy, since it generates the highest utility among the five options (16.42).

For our analysis, we list the optimal strategy for each scenario among the five possible options. We do impose a constraint where the utility values from the strategies that include an annuity must be at least 1% higher than the non-annuity option for the respective scenario. If they do not exceed that threshold, the non-annuity strategy is assumed optimal. For example, in Exhibit 5 for the 3% initial nominal withdrawal rate, while the non-annuity portfolio had a slightly lower level of utility than the annuity options (10.87 versus 10.88, respectively), since the difference is not deemed economically significant (using this threshold) the non-annuity option

			optimal w%		Initial Real Withdrawal Rate (%)				
eta	SS$	eq%	NO ANNUITY	WITH ANNUITY	2	3	4	5	6
1	$10	10	3.71	4.63	NoAnn	SPIA	PLIB	PLIB	PLIB
4	$10	10	2.85	3.78	NoAnn	DIA	PLIB	PLIB	PLIB
16	$10	10	2.00	2.87	DIA	PLIB	PLIB	PLIB	PLIB
1	$30	10	4.40	5.13	NoAnn	SPIA	PLIB	PLIB	PLIB
4	$30	10	3.53	4.34	NoAnn	SPIA	PLIB	PLIB	PLIB
16	$30	10	2.55	3.34	NoAnn	DIA	PLIB	PLIB	PLIB
1	$50	10	4.79	5.43	NoAnn	SPIA	PLIB	PLIB	PLIB
4	$50	10	3.90	4.65	NoAnn	SPIA	PLIB	PLIB	PLIB
16	$50	10	2.86	3.64	NoAnn	DIA	PLIB	PLIB	PLIB
1	$10	40	4.09	4.89	NoAnn	SPIA	SPIA	PLIB	PLIB
4	$10	40	3.07	3.92	NoAnn	SPIA	DIA	PLIB	PLIB
16	$10	40	2.18	2.99	DIA	DIA	PLIB	PLIB	PLIB
1	$30	40	4.89	5.44	NoAnn	SPIA	SPIA	PLIB	PLIB
4	$30	40	3.86	4.55	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$30	40	2.75	3.45	NoAnn	DIA	PLIB	PLIB	PLIB
1	$50	40	5.29	5.73	NoAnn	NoAnn	SPIA	PLIB	PLIB
4	$50	40	4.30	4.91	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$50	40	3.10	3.76	NoAnn	SPIA	PLIB	PLIB	PLIB
1	$10	70	4.20	5.15	NoAnn	SPIA	SPIA	PLIB	PLIB
4	$10	70	2.97	4.03	DIA	SPIA	PLIB	PLIB	PLIB
16	$10	70	2.02	3.07	DIA	DIA	PLIB	PLIB	PLIB
1	$30	70	5.25	5.82	NoAnn	SPIA	SPIA	SPIA	PLIB
4	$30	70	3.92	4.74	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$30	70	2.64	3.52	DIA	DIA	PLIB	PLIB	PLIB
1	$50	70	5.84	6.15	NoAnn	SPIA	SPIA	SPIA	PLIB
4	$50	70	4.48	5.16	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$50	70	3.03	3.85	DIA	SPIA	PLIB	PLIB	PLIB
		Average	**3.65**	**4.40**					

	Initial Real Withdrawal Rate (%)				
	2	**3**	**4**	**5**	**6**
NoAnn	21	1	0	0	0
SPIA	0	18	10	2	0
DIA	6	7	1	0	0
GLWB	0	0	0	0	0
PLIB	0	1	16	25	27

EXHIBIT 6. *Optimal Strategy for Real Withdrawal Rates*

would be deemed the optimal strategy. This "hurdle" is imposed to reflect the fact that if strategies result in utility values that are substantially similar, a household would be more likely to select the one that does not include an annuity.

While we are primarily focused on the relative efficiency of the strategies tested, in most instances each of the strategies that include an allocation to an annuity is better than the one that does not. In other words, any of the four strategies are generally better than not annuitizing. Therefore, while we effectively assume a household would be indifferent between GLWBs, PLIB, SPIAs, and DIAs, if the household is only willing to entertain certain strategies (e.g., a DIA) doing so would likely be better than not annuitizing, consistent with decades of research on this topic.

THE OPTIMAL STRATEGY

In this section we explore the relative efficiency of the five potential retirement income strategies considered. Opposed to selecting a potential single representative set of attributes we vary the assumed attributes to approximate different households for robustness purposes. Our analysis considers 135 different potential scenarios with the key attributes varying by risk aversion level, Social Security retirement benefits, portfolio allocations, and the initial nominal withdrawal rate.

Exhibit 6 includes the optimal strategy for real withdrawal rate for the five strategies considered. Note, the average optimal safe initial withdrawal rate (considering all strategies and potential withdrawal levels) was 4.4%, which is why the 4% is the middle initial real withdrawal rate selected.

There are a number of notable takeaways from Exhibit 6. First, the optimal initial real withdrawal rates differ significantly by scenario. For example, a household with a very high level of risk aversion, that does not purchase an annuit, with $10,000 in assumed Social Security benefits, and a 10% portfolio equity allocation would have a 2.00% initial real withdrawal rate. In contrast, another household that also does not want to annuitize, but with a very low level risk aversion, with $50,000 in assumed Social Security benefits, and a 70% portfolio equity allocation should have an initial real withdrawal rate of 5.84%. This is a staggering difference, especially since the assumed portfolio value is the same for both scenarios. This highlights a key weakness of using metrics like the probability of success to determine initial safe withdrawal rates because they generally cannot appropriately consider things like the magnitude of failure during retirement

Second, optimal initial withdrawal rates increased when considering an annuity, from 3.65% to 4.40% on average, but to varying degrees across household scenarios. The benefits of allocating to an annuity tended to be higher for households with higher risk aversion levels andlower levels of existing guaranteed income (i.e., Social Security retirement benefits). The impact of these is intuitive, since a household that is more risk averse with respect to an income shortfall would benefit more from transferring the longevity risk income component versus one that is more risk tolerant. The impact of guaranteed income relates to the potential impact to consumption if the portfolio is depleted. Households with relatively higher levels of guaranteed income level rely less on income from the portfolio to fund consumption and therefore are less affected should the portfolio become depleted.

Third, the general optimal annuitization strategy differs notably by withdrawal rate. For the lowest initial withdrawal rates (e.g., 3% nominal) not annuitizing or considering a SPIA or DIA would be optimal, assuming the household was comfortable with the irrevocable nature of the products, as the withdrawal rate increases, the PLIB becomes the most attractive while the GLWB was never optimal for any of the scenarios considered.

The results focusing on optimal initial nominal withdrawal rates, which are included in Appendix 2, are relatively similar to Exhibit 6. The SPIA does slightly better when focusing on nominal withdrawal rates, which is consistent with the fact the annuity provides a guaranteed (and fixed) nominal benefit.

The optimality of the PLIB compared to the SPIA and DIA seems like a "free lunch" to some extent, since they generate similar levels of certainty-equivalent income and do not require an irrevocable election. We provide some context around the potential drivers of this effect in a future section, but the differences can likely be attributed to a variety of factors, such as lapsation, mortality experience differences, and differences in return expectations (in particular the ability of the PLIB to access the positive assumed equity risk premium).

OPTIMAL EQUITY ALLOCATION OF THE PLIB

The base analysis assumes the PLIB is invested in a 40% equity portfolio. This is obviously a simplifying assumption and the actual optimal risk level for the PLIB is likely to vary by scenario (i.e., investor). While in reality there may be limits on risk levels within the PLIB (this is incredibly common with GLWBs) and/or the effective cost of the strategy could vary by portfolios (e.g., riskier portfolios have higher rider fees), it is worth exploring how the optimal equity allocation within the PLIB would vary across scenarios.

The analysis determines the optimal equity allocation for the same scenarios included in Exhibit 6 and the results are for the optimal equity allocations for real withdrawal rates are included in Exhibit 7 and the optimal equity allocations for nominal withdrawal rates are included in Appendix 3. We do not vary the fee for the PLIB by equity allocation, since it's not necessarily clear how the fee should vary (and some products have

		portfolio	Initial Real Withdrawal Rate (%)				
eta	**SS$**	**eq%**	**2**	**3**	**4**	**5**	**6**
1	$10	10	31%	55%	68%	80%	87%
4	$10	10	31%	50%	60%	68%	73%
16	$10	10	32%	51%	61%	64%	64%
1	$30	10	32%	57%	73%	88%	98%
4	$30	10	31%	54%	68%	79%	86%
16	$30	10	30%	50%	63%	70%	72%
1	$50	10	32%	57%	74%	90%	101%
4	$50	10	32%	56%	70%	84%	92%
16	$50	10	31%	51%	64%	73%	77%
1	$10	40	2%	17%	45%	63%	75%
4	$10	40	2%	20%	44%	57%	66%
16	$10	40	2%	32%	51%	59%	62%
1	$30	40	2%	16%	46%	69%	86%
4	$30	40	2%	17%	46%	63%	75%
16	$30	40	2%	24%	49%	62%	67%
1	$50	40	2%	16%	47%	72%	89%
4	$50	40	2%	17%	46%	67%	81%
16	$50	40	2%	21%	48%	62%	70%
1	$10	70	0%	0%	23%	47%	61%
4	$10	70	0%	0%	30%	47%	57%
16	$10	70	0%	20%	41%	52%	56%
1	$30	70	0%	0%	21%	50%	70%
4	$30	70	0%	0%	26%	49%	63%
16	$30	70	0%	6%	37%	52%	61%
1	$50	70	0%	0%	20%	51%	73%
4	$50	70	0%	0%	23%	49%	66%
16	$50	70	0%	0%	33%	51%	62%
		Average	**11%**	**25%**	**47%**	**64%**	**74%**

EXHIBIT 7. *Optimal PLIB Equity Allocations Across Household Attributes, Real Withdrawal Rates*

a constant fee), although we plan to explore this topic in future research.

There are two notable relationships given the optimal equity allocations in Exhibit 7. First, the optimal equity allocation for the PLIB increases for higher initial withdrawal rates. The effect is relatively monotonic and is likely due to the fact portfolios with higher equity allocations have higher expected returns and therefore create the higher probability of achieving a more aggressive equity target. If we assume the retiree household is using a 4% real withdrawal rate, which is approximately optimal, the average equity allocation would be approximately 50%.

Second, there is a clear negative relation between the equity allocation for the non-annuity portfolio and the optimal PLIB allocation. In other words, the PLIB allocation decreases (increases) as the portfolio equity allocation increases (decreases). This is somewhat counter to optimal equity allocations for GLWB portfolios, where the most aggressive portfolio would generally be considered optimal given the put option nature of the income.

PANEL A: NOMINAL SPENDING

PANEL B: REAL SPENDING

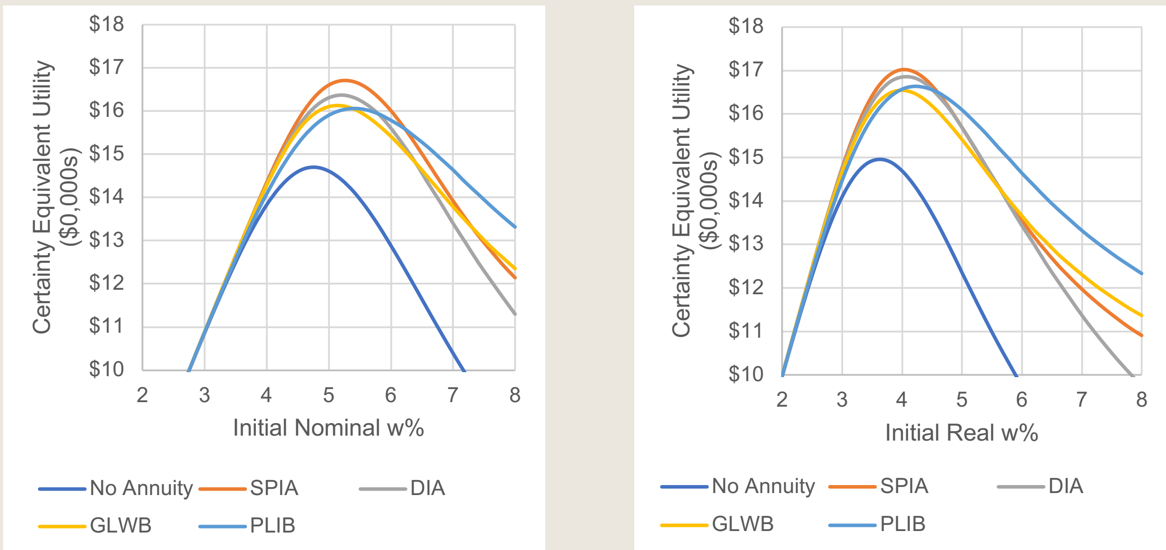

EXHIBIT 8. *Utility of Various Initial Withdrawal Rates Across Strategies, Equity Return Reduction of 2%*

Overall, this analysis suggests that more balanced risk strategies in PLIBs are likely to be optimal (e.g., 50% equities), although the ability to personalize the risk level is clearly important based on the household situation and preferences.

IMPACT OF LOWER RETURNS ON PLIBS

A key driver of the income for a PLIB is the positive assumed equity risk premium. The analysis assumes stocks outperform bonds annually by 5% (8.5% versus 3.5%, respective) on an arithmetic basis, and by approximately 3.5% on a geometric basis (based on the 18% and 7% assumed volatility levels, respectively).

PLIB strategies are going to be impacted more by future potential lower returns than other strategies that are more fixed in nature, such as SPIAs and GLWBs.[8] We demonstrate this effect in Exhibit 8, which uses the same underlying scenarios in Exhibit 3, but where we assume the arithmetic return on equities is reduced from 8.5% to 6.5% (i.e., by 2%).

We can see that in a lower return environment, the utility of all strategies decline, but the annuities that provide a more fixed guaranteed (i.e., SPIAs and GLWBs) experience less of a utility decline than those that are more subject to market returns (PLIBs and especially the non-annuity strategy). The PLIBs still do relatively well, though, especially for the highest withdrawal rate scenarios.

The relative gap between the non-annuity and annuity strategies widens, though, across all withdrawal rates where depletion is unlikely. In other words, lower returns make the all the annuities relatively more attractive than not annuitizing, although it does shi the relative value of the PLIB. It should be noted that the opposite is also true, and while lower returns impact the PLIB more than other strategies, higher returns would enhance the relative efficacy of a PLIB approach.

An important point when considering the potential income variation for the PLIB is that the income is in addition to an existing guaranteed income source: Social Security retirement benefits. This can significant-

8. While this is also technically true for DIAs, DIAs typically require less of a commitment than SPIAs, thereby leaving the portfolio more affected by market returns.

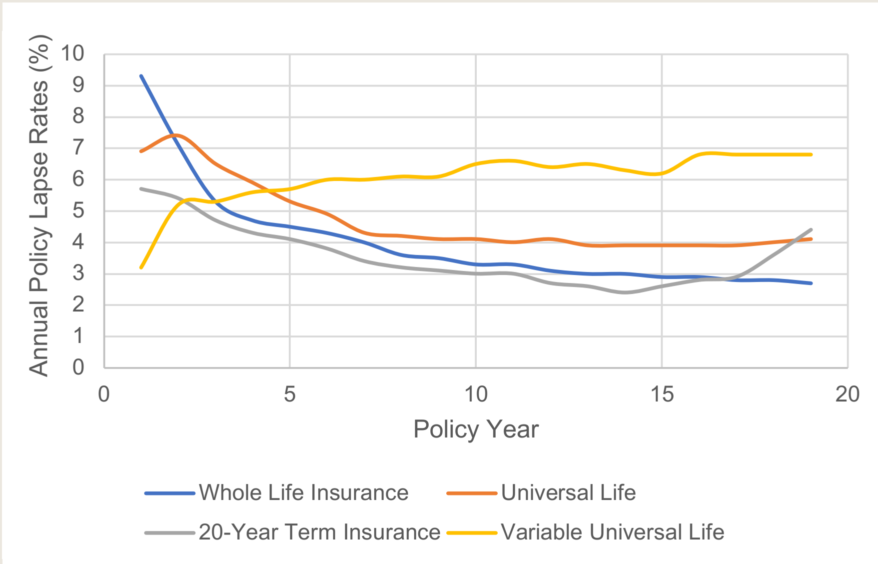

EXHIBIT 9. *Annual Policy Lapse Rates for Various Types of Life Insurance by Policy Year*

Source: *Society of Actuaries*

ly reduce the magnitude of a decline, especially if the household is willing to adjust spending based on market returns. The potential "adaptability" aspect of these strategies (with respect to withdrawals/spending) will be addressed in future research.

WHY DO GLWBS AND PLIBS LOOK SO ATTRACTIVE VERSUS SPIAS AND DIAS?

The relative efficiency of PLIB strategies versus SPIAs and DIAs may surprise some readers. In theory, the SPIA and DIA should always be more efficient than PLIBs (and GLWBs) since they require an irrevocable election (which is relatively painful for most households); however, this was clearly not the case for this analysis. There are a variety of factors that drive the relative noted benefits, in particular lapsation, mortality experience differences, and the equity risk premium (among others), which are discussed in this section.

LAPSATION

SPIAs and DIAs require an irrevocable election, whereby the annuitant cedes the premium for guaranteed income for life. The annuitant has no access to the fund, like they would for a GLWB and PLIB. This is both good and bad for the annuitant and good and bad for the insurance company.

Lapsation is common with types of insurance; for example, only roughly 50% of level term life insurance policyholders actually hold their policies to the end of the term period.[9] Exhibit 9 includes the lapse rates for various life insurance policies.

Lapse rates for individuals with GLWBs vary by a number of factors, but average 6.8% per year after a policy is out of its surrender period.[10] While some of this reflects poor decision making among annuitants/insured not utilizing the benefit (since they are paying for a benefit they don't end up utilizing), this isn't necessarily in all

9. https://www.soa.org/resources/research-reports/2019/2009-13-us-ind-life-persistency-update/
10. https://www.limra.com/en/research/benchmarks/VAGLBUS/Reporting/2018/GLWB

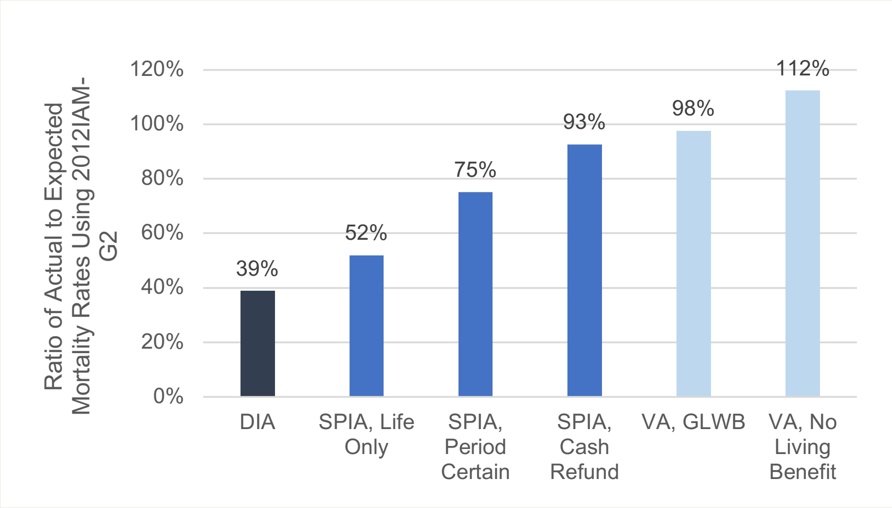

EXHIBIT 10. *Ratio of Actual to Expected Mortality Rate Experience for Various Annuity-Types*

Source: *Society of Actuaries, Ruark Consulting*

cases. For example, it gives the annuitant choice about how to most effectively utilize those assets versus other strategies that require an irrevocable election (e.g., an immediate annuity). It also provides an opportunity to "cash out" the policy and buy income at an even higher rate should the markets do well and/or interest rates rise.

MORTALITY EXPERIENCE

Mortality experience varies across annuities. For example, individuals who purchase life only annuities have significantly lower mortality rates (i.e., longer life expectancies) compared to annuities which provide some kind of minimum payment, as well as liquidity (i.e., variable annuities). This is not necessarily a surprise as only the healthiest individuals are likely to purchase an immediate annuity without any type of refund provision (this is the definition of adverse selection). Providing liquidity within the variable annuity structure requires significantly less commitment than a traditional immediate annuity behaviorally, and as a result the collective mortality rates of individuals who purchase these products tends to be higher (i.e., they have lower life expectancies than those who purchase life only annuities).

This effect is demonstrated in Exhibit 10, which includes data on mortality rate experience in the Society of Actuaries 2009-2013 Individual Payout Annuity Mortality Experience Report[11] for deferred and immediate annuities, and the Ruark 2018 Variable Annuity Industry Mortality Experience Study for variable annuities.

Exhibit 10 demonstrates the somewhat monotonic relationship between mortality experience and the "commitment" with respect to annuitization. DIAs perhaps reflect the most direct hedge against longevity risk and therefore it is not a surprise that these products have had the lowest mortality experience (i.e., individuals who buy DIAs tend to have higher life expectancies than individuals who buy other products).

The mortality rates for individuals who purchase VAs are clearly higher than other products (notably DIAs and SPIAs) and this can priced into the payouts of the approaches. In other words, by reducing the adverse selection associated with annuities, the implied payouts for products with GLWBs can be higher.

11. https://www.soa.org/resources/experience-studies/2016/2009-13-invidual-payout-annuity/

Percentile						
Country	**5th**	**25th**	**50th**	**75th**	**95th**	**Geomean**
Australia	-21.40	-2.61	5.78	13.94	36.13	4.75
Austria	-34.83	-12.28	1.24	19.08	109.71	4.39
Belgium	-26.50	-8.15	2.94	13.53	37.23	2.02
Canada	-23.89	-7.29	4.48	15.20	39.17	3.32
Denmark	-20.19	-7.45	4.79	17.04	35.18	3.36
Finland	-29.41	-6.92	4.69	22.91	51.27	5.24
France	-26.84	-9.53	2.75	15.04	48.55	2.92
Germany	-28.55	-6.85	4.69	20.08	58.81	4.48
Ireland	-26.22	-4.59	3.46	14.97	34.48	2.55
Italy	-30.97	-11.61	5.04	17.88	55.84	2.75
Japan	-31.12	-8.81	2.62	23.55	59.89	5.06
Netherlands	-28.31	-5.36	3.68	16.71	36.65	3.19
New Zealand	-18.90	-3.42	5.70	13.30	34.66	4.09
Norway	-22.99	-6.44	2.87	15.78	40.10	2.50
Portugal	-31.20	-6.51	5.45	20.83	52.22	5.06
South Africa	-21.46	-5.74	6.05	18.77	41.87	5.06
Spain	-29.72	-7.14	0.70	12.90	44.95	1.35
Sweden	-26.66	-6.24	6.03	17.00	41.45	3.21
Switzerland	-25.17	-5.12	3.30	14.26	33.51	2.19
UK	-21.06	-4.28	4.04	10.51	34.72	3.36
US	-7.77	-1.25	1.07	2.98	6.97	0.78
Average	**-25.39**	**-6.55**	**3.87**	**16.01**	**44.45**	**3.41**

EXHIBIT 11. *Equity Risk Premium vs Bonds: 1900 to 2020*

Source: *DMS dataset*

HIGHER POTENTIAL RETURNS

Using a variable annuity account structure, the annuitant can capture higher potential returns (e.g., more of the equity risk premium) than more conservative annuity payout strategies (e.g., an immediate annuity). While this potentially exposes the annuitant to slightly more downside risk, depending on the investment strategy, it also offers more upside if equities continue to outperform.

The outperformance of equities over fixed income has been relatively pronounced historically, both in the US and internationally. This effect is documented in Exhibit 11, which includes the distribution of the historical realized equity risk premium of stocks versus bonds from 1900 to 2020 based on the 21 countries[12] in the Dimson, Marsh, and Staunton data.

While past performance is no guarantee of future results, there is significant evidence that stocks have

12. Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, United Kingdom, and United States

outperformed bonds over the long-term. This gives the annuitant the potential to generate more income than would be invested in safer protected strategies, such as a SPIA.

CONCLUSIONS

The "annuity puzzle" is still alive and well in America today. There is a new guaranteed income product strategy that is gaining adoption in the market today, though, that appears to hold significant promise; an approach we refer to as a "protected lifetime income strategy" PLIB.

PLIBs are structurally similar to GLWBs, but there are notable differences in how the potential benefits can evolve during retirement. This research suggests the increased risk "sharing" approach with respect to PLIBs can benefit retirees, especially those targeting higher initial withdrawal rates (who are therefore more likely to benefit from annuitization).

PLIBs can exist in a variety of structures, such as overlaying a regular portfolio (e.g., as CDA), as well as within an annuity, and be combined with a variety of investment strategies, such as FIAs, VAs, RILAs, etc.

However, the most important objective is to get retirees to increase their allocations to guaranteed income. While PLIBs may be more efficient than other approaches (e.g., GLWBs), any of the annuity strategies considered is generally better than not annuitizing, and therefore retirees and financial advisors should determine the optimal strategy in light of the unique preferences of the client.

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APPENDICES

APPENDIX 1. *Annuity Allocation Model*

For each simulated income path, the utility-equivalent constant income level is calculated based on the elasticity of intertemporal substitution parameter, which is denoted as II. That is, for a given simulated income path, II is the constant amount of income with the same utility as the actual income path. This is given by

$$II = \left(\frac{\sum_{t=0}^{T} q_t (1+\rho)^{-t} I_t^{\frac{\eta-1}{\eta}}}{\sum_{t=0}^{T} q_t (1+\rho)^{-t}} \right)^{\frac{\eta}{\eta-1}} \qquad \text{[A1.1]}$$

Where I_t is the level of income in year t, q_t is the probability of surviving to at least year t, based on the Society of Actuaries 2012 immediate annuity mortality table, T is the last year for which $q_t>0$, and ρ is the investor's subjective discount rate.

The value of the potential bequest is denoted along path i at time t, B_{it}. Above the probability of surviving is defined to at least year t as q_t . So the probability of dying in year t is $q_t - q_{t+1}$. These probabilities are used together with the subjective discount rate to calculate a weighted average bequest for each path i:

$$\bar{B}_i = \frac{\sum_{t=0}^{T} (q_t - q_{t+1})(1+\rho)^{-t} B_{it}}{\sum_{t=0}^{T} (q_t - q_{t+1})(1+\rho)^{-t}} \qquad \text{[A1.2]}$$

$\bar{B}_i$ and II_i are combined to form a measure of the utility of path i in the same units as income. Since IIi is the constant level of income that has the same utility as the actual path of income, it can be expressed as a lump sum (the discounted value of the income stream) at time 0 by multiplying it by

$$\Delta = \sum_{t=0}^{T} q_t (1+\rho)^{-t} \qquad \text{[A1.3]}$$

Therefore $\bar{B}_i$ can be converted to an equivalent constant level of income by dividing it by Δ. To translate $\bar{B}_i/\Delta$ into the incremental benefit of the possibility of leaving a bequest in addition to the stream of income under path i, the parameter τ is introduced, which measures the strength of the bequest motive. Hence the constant level of income that is equivalent to the income path together with the possible bequests of each year is $II_i + \tau\bar{B}_i/\Delta$.

The expected utility is measured using the CRRA utility function with its risk tolerance parameter θ that was introduced in equation 1:

$$EU = \sum_{i=1}^{M} p_i \frac{\theta}{\theta-1} (II_i + \tau\bar{B}_i/\Delta)^{\frac{\theta-1}{\theta}} \qquad \text{[A1.4]}$$

where M is the number of paths, the subscript i denotes which of M paths is being referred to, and p_i is the probability of path i occurring which is set to $1/M$. Y is defined as the constant value for II that will yield this level of expected utility. This is the certainty-equivalent of the stochastic utility-adjusted income II. Y is given by

$$Y = \left[\sum_{i=1}^{M} p_i (II_i + \tau\bar{B}_i/\Delta)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \qquad \text{[A1.5]}$$

The optimal strategy would be the one that maximizes the value of Y.

			optimal w%		Initial Nominal Withdrawal Rate (%)				
eta	SS$	eq%	NO ANNUITY	WITH ANNUITY	3	4	5	6	7
1	$10	10	4.91	5.91	NoAnn	SPIA	SPIA	PLIB	PLIB
4	$10	10	3.97	5.02	NoAnn	SPIA	PLIB	PLIB	PLIB
16	$10	10	3.11	4.30	SPIA	DIA	PLIB	PLIB	PLIB
1	$30	10	5.62	6.45	NoAnn	NoAnn	SPIA	PLIB	PLIB
4	$30	10	4.72	5.59	NoAnn	SPIA	PLIB	PLIB	PLIB
16	$30	10	3.66	4.64	NoAnn	SPIA	PLIB	PLIB	PLIB
1	$50	10	6.01	6.75	NoAnn	NoAnn	SPIA	PLIB	PLIB
4	$50	10	5.11	5.93	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$50	10	4.00	4.90	NoAnn	SPIA	PLIB	PLIB	PLIB
1	$10	40	5.29	6.16	NoAnn	SPIA	SPIA	PLIB	PLIB
4	$10	40	4.19	5.16	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$10	40	3.25	4.45	SPIA	DIA	PLIB	PLIB	PLIB
1	$30	40	6.21	6.83	NoAnn	NoAnn	SPIA	SPIA	PLIB
4	$30	40	5.06	5.79	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$30	40	3.85	4.77	NoAnn	SPIA	DIA	PLIB	PLIB
1	$50	40	6.68	7.17	NoAnn	NoAnn	SPIA	SPIA	PLIB
4	$50	40	5.53	6.19	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$50	40	4.23	5.03	NoAnn	SPIA	SPIA	PLIB	PLIB
1	$10	70	5.34	6.43	NoAnn	SPIA	SPIA	SPIA	PLIB
4	$10	70	3.97	5.25	DIA	SPIA	SPIA	PLIB	PLIB
16	$10	70	2.92	4.55	DIA	SPIA	PLIB	PLIB	PLIB
1	$30	70	6.54	7.21	NoAnn	SPIA	SPIA	SPIA	PLIB
4	$30	70	5.06	5.96	NoAnn	SPIA	SPIA	PLIB	PLIB
16	$30	70	3.61	4.84	DIA	SPIA	SPIA	PLIB	PLIB
1	$50	70	7.27	7.61	NoAnn	SPIA	SPIA	SPIA	SPIA
4	$50	70	5.64	6.44	NoAnn	SPIA	SPIA	SPIA	PLIB
16	$50	70	4.05	5.10	DIA	SPIA	SPIA	PLIB	PLIB
		Average	**4.81**	**5.72**					

	Initial Nominal Withdrawal Rate (%)				
	3	**4**	**5**	**6**	**7**
NoAnn	21	4	0	0	0
SPIA	2	21	19	6	1
DIA	4	2	1	0	0
GLWB	0	0	0	0	0
PLIB	0	0	7	21	26

APPENDIX 2. *Optimal Initial Nominal Withdrawal Rate*

			Initial Nominal Withdrawal Rate (%)				
eta	**SS$**	**eq%**	**3**	**4**	**5**	**6**	**7**
1	$10	10	21%	48%	57%	68%	77%
4	$10	10	21%	45%	53%	61%	67%
16	$10	10	23%	46%	52%	56%	60%
1	$30	10	21%	48%	60%	73%	86%
4	$30	10	21%	48%	57%	68%	77%
16	$30	10	21%	46%	54%	61%	66%
1	$50	10	21%	49%	61%	75%	89%
4	$50	10	21%	48%	59%	71%	82%
16	$50	10	21%	46%	55%	64%	70%
1	$10	40	4%	12%	33%	52%	65%
4	$10	40	4%	15%	35%	50%	59%
16	$10	40	4%	21%	40%	50%	54%
1	$30	40	3%	11%	32%	55%	73%
4	$30	40	4%	13%	33%	53%	66%
16	$30	40	4%	17%	39%	51%	59%
1	$50	40	3%	11%	32%	56%	75%
4	$50	40	4%	12%	33%	54%	69%
16	$50	40	4%	16%	37%	52%	62%
1	$10	70	0%	0%	13%	37%	53%
4	$10	70	1%	0%	22%	40%	51%
16	$10	70	0%	12%	30%	42%	49%
1	$30	70	0%	0%	8%	39%	58%
4	$30	70	0%	0%	15%	39%	55%
16	$30	70	1%	1%	28%	42%	52%
1	$50	70	0%	0%	7%	39%	60%
4	$50	70	0%	0%	12%	39%	57%
16	$50	70	0%	0%	24%	42%	53%
		Average	**8%**	**21%**	**36%**	**53%**	**65%**

APPENDIX 3. *Optimal PLIB Equity Allocations Across Household Attributes, Nominal Withdrawal Rates*