WHO SHOULD PURCHASE VARIABLE ANNUITIES AND HOW SHOULD THEY USE THEM?

LEORA FRIEDBERG, GUOXUAN MA, WEI SUN AND ANTHONY WEBB

ABSTRACT

Prominent among the financial risks that retirees face are longevity and investment risk. Immediate annuities provide almost complete insurance against both risks, but take-up is extremely low. We focus on a simple explanation for this annuity puzzle: the annuity products that households actually buy may offer a more appealing trade-off between risk and return than that offered by immediate annuities. Some households may be better off if, at the time of retirement, they purchase the partial insurance against investment and longevity risk provided by variable deferred annuities with a guaranteed living withdrawal benefit (GLWB) rider. We construct a model of optimal post-retirement consumption, asset allocation, and bequest choices. We find that a bequest motive reduces but does not eliminate annuity demand and, in fact, because the variable annuity allows individuals to enjoy the equity premium while managing both investment and longevity risk through the GLWB option, they dominate immediate annuities. The optimal allocation to risky assets for someone who purchases a variable annuity will be high. Incorporating taxes has little effect for most, while registered index-linked annuities can improve outcomes relative to no annuitization, depending on the terms of the contract and the household's beliefs and preferences.

INTRODUCTION

Prominent among the financial risks that retirees face are longevity and investment risk. One type of annuity, termed an immediate annuity, provides mostly complete insurance against both risks: in exchange for a lump sum of wealth, a purchaser receives a lifetime income, fixed in nominal terms.[1] Theoretical calculations indicate that, inflation risk apart, the insurance provided by immediate annuities ought to be valued by risk-averse households facing an uncertain lifespan and uncertain investment returns (Friedberg and Webb 2022). However, take-up of immediate annuities in the private market is extremely low – which is the so-called annuity puzzle. In 2022, fixed immediate annuity sales in the United States totaled a mere $5.9 billion.[2]

The academic literature attempts to resolve the annuity puzzle by positing either a range of possible behavioral biases (for example, that households overvalue lump sums relative to income streams) or else a range of possible gaps in theoretical models that may reduce the value of annuities (for example, a luxury bequest motive, as in Lockwood, 2012).[3] While those factors might well influence demand for annuitization, we focus on a simpler explanation: that the annuity products that households actually buy may offer a more appealing trade-off between risk and return than that offered by immediate annuities. The annuity products that are most common in the market are variable deferred and, more recently, fixed index and registered index-linked annuities (which we collectively refer to as variable annuities) and can include a rider providing an option to convert some of

1. The insurance is not quite complete because, following the withdrawal of inflation indexed immediate annuities from the market, the individual is exposed to inflation risk.
2. U.S. Individual Annuity Sales Surveys, LIMRA Secure Retirement Institute. https://www.limra.com/siteassets/newsroom/fact-tank/sales-data/2023/q2/2q-2023-annuity-sales-estimates-v-final.pdf
3. Many of these explanations have been reviewed in Webb (2021a, 2021b).

the asset into lifetime income.[4] This may explain why variable annuity sales totaled $102.9 billion and fixed index annuities $79.8 billion in 2022, though the academic literature often ignores those real-world products.

In contrast to an immediate annuity, a variable deferred annuity with a Guaranteed Lifetime Withdrawal Benefit (GLWB) rider functions as an investment vehicle (the variable annuity) with a *lifetime income option* (the rider), as the full taxonomy in Friedberg and Webb (2022) explains.[5] The premium (that is, the amount used to purchase the annuity product) is invested in financial assets, typically stock and bond funds, and the policyholder enjoys the returns on those funds, minus fees and expenses. Registered index-linked annuities, which have become increasingly popular in the annuity market, further allow individuals to limit their exposure to market risk by choosing pre-set floors and/or ceilings for returns.[6] Attached to any of these products, a GLWB rider gives the policyholder the right, but not the obligation, to commence taking annual withdrawals from their invested assets at a date of their choosing.[7] The assets continue to earn market returns for as long as the assets remain, and in the event that GLWB withdrawals deplete the assets, whether because policyholder lives longer than expected or investment returns have been poorer than expected, the insurance company steps in and makes the GLWB payments for the remainder of the policyholder's life.

The insurance provided by an immediate annuity comes at a cost not only of a loss of liquidity and reduction in bequest size but also of access to the higher expected returns (relative to alternatives) that continue to be offered by equities – and for plausible preference parameters, this demand for equities is higher for individuals with greater Social Security income as a share of their retirement resources, because Social Security as an asset has investment characteristics similar to those of low-risk bonds.[8] On the other hand, the no-arbitrage condition means that the GLWB benefit must be less than the income provided by an immediate annuity, absent possible differences in mortality rates between purchasers of immediate and variable annuities. Therefore, less risk-averse households may be better off if, at the time of retirement, they purchase the partial insurance against investment and longevity risk provided by variable deferred annuities with a guaranteed living withdrawal benefit (GLWB) rider, rather than purchasing an immediate annuity of any size. This hypothesis is independent of the preferred tax treatment of variable annuity assets, which has its greatest value if purchased before retirement, when households face higher marginal tax rates than those anticipated post-retirement. The most catastrophic financial outcome for households that begin retirement with means is to both live longer than expected and experience poor investment returns, and it is this combination of outcomes that a variable annuity with a GLWB rider insures against.

The academic literature on annuities, while rich in some dimensions, is sparse in other important ones. Numerous academic papers have used numerical methods to theoretically model the value of immediate annuities to risk-averse households seeking to manage wealth decumulation in retirement (Yaari 1965, Mitchell, Poterba, Warshawsky, and Brown 1999, Brown and Poterba 2000, Dushi and Webb 2004, Lockwood 2012). However, only a few have applied those methods to investigate the financial value of the annuity products that dominate the annuity market (Milevsky and Salisbury 2006, Xiong, Idzorek, and Chen 2010, Huang, Milevsky, and Salisbury 2014, Moenig and Bauer 2015), and fewer go

4. All annuity products are required to offer an option to convert the annuity into lifetime income. This option is rarely exercised and is distinct from the GLWB rider. With the latter, the payments are funded by liquidating the annuity investments, with the insurance company making up the shortfall should the annuitant outlive their capital.
5. VAs offer the option to purchase other riders (Pfau, 2019), insuring other risks. As our focus is on insuring longevity risk, we defer study of other riders to future research. Meanwhile, the evolution of annuity products continues as Blanchett (2023) emphasizes. The latest offerings, involving what he terms "Protected Lifetime Income Benefits", offer payouts that encompass more investment risk than those from GLWB riders.
6. They pay annual interest equal to some percentage of the return (excluding dividends) calculated on some stock market index—for example, the S&P 500—subject to floors and ceilings. A typical floor is zero percent; if the floor is less than zero, the annuity is technically a variable index annuity.
7. The exercise price of the GLWB option is a complicated function of both age and the "high-water mark" of the annuity value. We model this function carefully in our analysis. Should the household wish, the variable annuity itself can be surrendered for a lump sum.
8. In the presence of an equity premium, most households should still annuitize at least partially, because most households hold some of their wealth in bonds, and the annuity would substitute for bonds. Nor can the equity premium explain the lack of demand previously for variable immediate annuities, an immediate annuity that is no longer available, where the income is related to the return on an underlying stock fund.

beyond asset-pricing exercises to account for annuity valuation when individuals face incomplete insurance markets (Horneff, Maurer, Mitchell, and Rogalla 2013, Steinorth and Mitchell 2015). Moreover, among papers with numerical models of immediate annuities in incomplete markets, only Brown, Mitchell, and Poterba (2001) incorporates investment risk and the equity premium, key factors affecting both the value of real-world annuities and overall asset allocation.

Our approach builds on the general set of models used to evaluate immediate annuities and applies them to the more difficult problem of evaluating variable annuities. We construct a model of optimal post-retirement consumption and asset draw-down choices by a risk-averse individual. The individual faces an uncertain lifespan and uncertain stock market returns and decides whether to purchase a variable annuity (and the share of assets to devote to that purchase) with a GLWB or an immediate annuity that has realistic fees. We further incorporate asset allocation decisions, both inside and outside of the variable annuity; the decision of when to convert annuity assets into lifetime income; and the luxury bequest motive that, in Lockwood (2012), reduces annuity demand. We focus on someone at age 65 who is retired, since this is when many households make decisions about their post-retirement assets, and we focus on asset levels corresponding to the upper part (though not the top) of the asset distribution, since those are the households that stand to gain most from avoiding self-insurance. We then consider purchase decisions by men and women separately, which is important given women's greater longevity and unisex pricing of variable annuities (in contrast to immediate annuities purchased outside of employer sponsored retirement plans).

The goal of our model is to analyze in which situations individuals should optimally purchase a variable annuity (or registered index-linked annuity) with a lifetime income option; an immediate annuity; or no annuity. We determine this by calculating annuity-equivalent wealth, the factor by which the wealth of someone who is unable to purchase an annuity must be increased so that a risk averse individual is, in expectation, just as well off as the same individual with access to the annuity market.[9] To illustrate, if an individual had age-65 financial assets of $100,000 and the right to use up to (say) one-half of their wealth to purchase annuities, then annuity equivalent wealth of 1.2 implies that individual would be indifferent between $100,000 plus the right to purchase annuities and $120,000 without that right.[10] We consider annuity-equivalent wealth of variable and registered index-linked annuities with GLWB riders, along with immediate annuities. Those metrics likely understate the value of annuities because we assume that the alternative to annuitization is an optimal drawdown of unannuitized wealth, a strategy that few, if any, households can compute.[11]

The model also shows how the value of variable annuities is affected by other financial decisions of individuals, to provide guidance on those decisions to purchasers and their financial advisors. The decisions facing the purchaser of a variable annuity are complex. As with the purchaser of an immediate annuity, the individual must decide how much of their wealth to invest in the variable annuity. An individual who invests a large share of their wealth in the variable annuity and takes non-GLWB withdrawals from the variable annuity to finance consumption will reduce the amount of their GLWB guarantees and in effect waste some of their GLWB premiums. The individual must also decide how to invest both variable annuity and non-variable annuity wealth. The insurance provided by the GLWB against bad investment outcomes may incentivize the individual to increase the riskiness of the variable annuity portfolio, subject to insurance company limits, and perhaps reduce the riskiness of the non-variable annuity portfolio.

The optimal exercise decision of the GLWB is also complex. An individual who postpones exercising the GLWB

9. To be clear, this is a statement about welfare, not expected wealth. Given assumed preferences and beliefs, the expected utility of the individual must be equal in each scenario.
10. By design, annuity equivalent wealth can never be less than zero as the individual can decline the annuitization option. Our model calculates negative annuity equivalent wealth for such households by requiring them to annuitize.
11. In practice, households appear to follow arbitrary rules of thumb such as spending interest and dividends only. Only by chance will interest and dividends correspond to the optimal share of wealth to consume. Alternatively, they may follow the well-known 4% rule, which fails to respond to realized returns and therefore risks complete immiseration (Friedberg and Webb 2022).

option increases the amount of their GLWB income because the income as a percent of the high-water mark of the contract value (that is, the highest value of the underlying stock market or other index, adjusted for excess withdrawals) is higher at older GLWB exercise ages. Delaying exercise may also increase the likelihood of hitting a new high-water mark, especially if the account value is close to a previous high-water mark. So, an individual who delays exercise receives a larger monthly or annual payment but for a shorter period. However, a strategy of choosing an exercise age to maximize the expected present value of lifetime income neglects the value of the additional longevity insurance purchased because of delay. As with the Social Security claiming decision (an even more advantageous annuity option, as Sun and Webb 2011 demonstrate), an individual who delays is, in effect, using the GLWB payments foregone to purchase additional longevity insurance.

We find that at least partial annuitization, whether through an immediate annuity or a variable annuity with a GLWB rider, dominates no annuitization, and more so at higher levels of risk aversion. We find that a bequest motive reduces but does not eliminate annuity demand, unlike in Lockwood (2012), which has a model without investment risk or the equity premium.[12] The upshot is that variable annuities with a GLWB option dominate not only no annuitization but also immediate annuities, no matter the level of an individual's risk aversion that we consider, because the variable annuity allows individuals to benefit from the equity premium while managing both investment and longevity risk

When we consider further details, we find that for typical variable annuities, it is optimal to exercise the GLWB option immediately, at age 65. The additional longevity insurance acquired because of delay is insufficient to compensate for delaying receipt while still enjoying the equity premium, and we explore terms under which it becomes optimal to delay.[13] This is the case even though delayed exercise increases the amount of expected lifetime income. The optimal asset allocation within the variable annuity depends on the assumed level of the equity premium, and for plausible assumed levels, it will be optimal to select the largest permitted allocation to risky assets. Moving away from this optimal allocation, however, can substantially reduce the value of variable annuities, an important result for financial advisors to keep in mind. Holding benefits constant, the level of fees has a significant effect on the value of variable annuities, and therefore the ranking of annuity options (considering both type and amount) depends on preferences, beliefs, and the level of fees and benefits – and we caution that variable annuity fees and benefits differ among providers, which makes it difficult to lay down general rules. Incorporating taxes generally has little effect on the optimal strategy because only the wealthiest retirees pay significant amounts of income tax after retirement.[14] We find that registered index-linked annuities can increase expected household financial well-being, depending on the terms of the contract and the individual's beliefs and ability and willingness to bear risk.

The remainder of the paper proceeds as follows. The first section surveys the annuity literature. The second section presents our model. The third section presents results, and the fourth section discusses implications for households and their advisors.

2. BACKGROUND

We refer readers to Pfau (2019) for an excellent summary of the various types of annuities and to Friedberg and Webb (2022) for a discussion of the simple economics

12. Lockwood (2018) estimates bequest function parameters based on a model with longevity and long-term care risk and analyzes demand for long-term care insurance. While our model does not incorporate long-term care risk, the analysis in Lockwood (2012) demonstrates that demand for annuities is largely unaffected by incorporating long-term care risk.
13. This finding echoes similar results in Sun and Webb (2011). They show that delaying Social Security claiming past age 68 is not optimal, even at higher age-related increases until age 70 than is offered by GLWB riders; their result, however, depends on having a single risk-free asset, so our finding here derives from a more general setting.
14. Variable annuities offer the same tax treatment as tax-deferred retirement accounts. Therefore, they do not confer tax advantages if purchased within a tax-deferred account, and their tax advantages largely arise if purchased pre-retirement, which both lengthens the tax deferral period and allows the shifting of taxable income to a time period when marginal tax rates may be lower. However, effective marginal tax rates of retirees are a complicated function of income because of tax provisions affecting combined-income phase-out rules and Medicare income-tested premiums, potentially reducing the gains from deferral in retirement.

of each. Here, we discuss how findings in the extensive literature relating to immediate annuities apply to variable annuities. We then review the sparse literature relating to variable and fixed indexed annuities.

IMMEDIATE ANNUITIES

The classic paper demonstrating when full annuitization is optimal, by Yaari (1965), has a very simple life-cycle model in which risk-averse individuals get time-separable utility from consumption and face lifespan uncertainty. Once any additional sources of uncertainty are added or preferences are assumed to be more complicated, then it is necessary to use numerical optimization techniques to calculate the value of annuities. The models mostly assume away investment risk and the possibility of earning an equity premium and mostly assume an all-or-nothing option to purchase an annuity at retirement. Abstracting from investment risk may not be too consequential if individuals have access to both fixed and variable immediate annuities but matters if only fixed immediate annuities are available because the individual must trade off the purchase of longevity insurance against the loss of the equity premium. Confronting the household with an all-or-nothing decision may understate the value of annuities because, when annuities are actuarially unfair (that is, the expected present value of the income stream, discounted by an interest rate and annual survival probabilities is less than the premium paid inclusive of fees), it may be optimal for households to delay annuitization and annuitize only part of their wealth (Dushi and Webb, 2004).

Early papers (Mitchell, Poterba, Warshawsky, and Brown, 1999, Brown and Poterba, 2000, Davidoff, Brown, and Diamond 2005) concluded that, although immediate annuities were actuarially unfair to households with population average mortality, most households would be better off annuitizing. These papers likely understate the value of annuitization because individuals with annuitizable wealth have lower-than-population average mortality (Dushi and Webb, 2006). Those findings led to a series of papers seeking both behavioral (Brown, 2007) and rational explanations (Lockwood, 2012) for this so-called annuity puzzle. One of our goals here it to see whether we can help resolve the annuity puzzle without resorting to behavioral explanations, in which households exhibit choice inefficiencies. An important example of this is Lockwood (2012), who argues that a plausible specification of the bequest motive, together with observed levels of actuarial unfairness, is sufficient to eliminate the demand for immediate annuities. We find that the Lockwood result depends critically on the assumption that the beneficiary has lower consumption and thus a higher marginal utility of consumption than that obtained from the Social Security benefits of the household leaving the bequest.

VARIABLE ANNUITIES

A striking feature of the academic annuitization literature is the frequent neglect of the annuities households actually buy, namely variable and fixed indexed annuities. Our reading of the sparse literature on the value of variable annuities in managing post-retirement wealth decumulation is that it fails to deal fully with the options available upon purchase. One branch of the literature prices the complex options embedded in the products (Milevsky and Salisbury, 2006, Moenig and Bauer, 2015), but their findings tell us little about the value of the product to a household who, in the absence of any alternative hedging mechanism, may value the product more highly than indicated by option pricing techniques.

A second branch considers the value of the products to risk-averse households. Horneff, Maurer, Mitchell, and Rogalla (2013) focus on the use of a guaranteed minimum withdrawal benefit (GMWB) rider during the pre-retirement period. A GMWB rider guarantees that a policyholder can withdraw a periodic amount equal to the premium paid (for example five percent of the premium for 20 years) but does not guarantee a lifetime benefit. It is therefore a less effective means of hedging longevity risk than a GLWB rider. Steinorth and Mitchell (2015) consider the use of GLWB riders during the post-retirement period. They assume that the household must choose between investing all or none of its wealth in the variable annuity. We find that allocating part of the household's wealth to the variable annuity is often optimal, so their assumption will understate the value of variable annuities. Xiong, Idzorek, and Chen (2010) use option pricing theory to price GLWB riders. The authors show that the put option embedded in the

GLWB enables the household to tolerate more risk in the rest of its portfolio, but only over longer time horizons. We also find that variable annuities permit greater risk-taking. In the absence of the variable annuity and assuming no bequest motive, households with high levels of risk aversion will allocate 54% of their financial assets to stocks at age 65. When the variable annuity is available, households will optimally put all the wealth into the variable annuity account and allocate all of it to stocks.

In sum, none of the papers in this literature investigate the value of variable annuities as a drawdown tool, nor provide guidance as to how households can use the options embedded in variable annuities to best advantage.

REGISTERED AND FIXED INDEXED ANNUITIES

We are unaware of any literature that uses numerical techniques to calculate annuity equivalent wealth for registered or fixed index annuities, Moenig (2021) discusses how registered index linked annuities are structured and how insurers may price and hedge them.

3. MODEL

Our model of utility-maximizing individuals builds on the papers studying demand for immediate annuities, while incorporating several key features that are critical for understanding the value of variable annuities. Our model is most closely related to those of Mitchell, Poterba, Warshawsky, and Brown (1999), Brown and Poterba (2000), Dushi and Webb (2004), and Lockwood (2012), and though it does not consider married couples, as Brown and Poterba and Dushi and Webb do, it adds a flexible bequest motive as Lockwood does. To understand how individuals should optimally use variable annuities, it also adds asset allocation decisions both in and out of the variable annuity, along with the optimal exercise age of the GLWB.

3.1 SETUP

We begin by considering a single male aged 65 who is retired and has the annual mortality risk of annuity purchasers, and then later we do the same for a female.[15] The individual receives $26,000 annually from Social Security ($2,167 a month), the average amount for new retired worker benefit claimants.[16] The individual also has $400,000 in retirement accounts that may be used to purchase an annuity.[17] This wealth level is higher than the median and reflects our focus on individuals who possess sufficient financial wealth for that wealth to contribute meaningfully to retirement financial security but not so much that the household faces negligible risk of outliving its wealth. The individual has a time-separable constant relative risk aversion (CRRA) utility function, as is standard in the literature, with a risk aversion coefficient of either two or five, exponential time discounting with a discount rate of 3%, and varying strengths of bequest motive.[18] Excluding bequests, the individual's objective is to maximize his expected discounted lifetime utility:

$$maxE_t(\sum_{t=65}^{t=T} \beta^t \rho_t U(C_t)) \quad (1)$$

where t is age, the maximum possible survival age T is 115, β is the time discount factor, ρ_t is the probability of surviving to time t, and $U(C_t)$ is the utility of consumption at time t. We do not model health and long-term care cost risk nor allow the marginal utility of consumption to vary with age or health status.[19]

15. We use mortality rates reported in the Society of Actuaries Annuity 2000 mortality table, projected using Projection Scale AA to yield mortality rates for a male or female born 1958.
16. Table 6.B3 Social Security Administration Annual Statistical Supplement 2022, increased by inflation to 2023 and rounded.
17. As a point of reference, retirement account balances in 2019 for households aged 65-74 had a mean value of $494,000 and a median value of $190,000 (The Fed - Table: Survey of Consumer Finances, 1989 - 2022 (federalreserve.gov)).
18. Estimates of the coefficient of risk aversion tend to cluster between 2 and 10 depending in part on whether the estimates are derived from portfolio theory, purchases of insurance, economic experiments, or preferences over lotteries (Chetty 2006).
19. This approach maintains comparability with the rest of the annuity literature. While incorporating health risk is of interest and has been explored in models without annuitization by De Nardi, French and Jones (2010), it makes the model computationally infeasible given current computing limits.

The individual makes the following choices:

1. At age 65, how to allocate financial assets between an annuity product and non-annuity wealth.[20]
2. Annually, how to allocate variable annuity wealth between risky stock and a risk-free bond,
3. Annually, how to allocate non-variable annuity wealth between risky stock and a risk-free bond,
4. Annually, the amount to be consumed from non-variable annuity wealth,
5. Annually, the amount to be consumed through a non-GLWB withdrawal from the variable annuity, and
6. At ages 65, 70, 75, and 80, whether to exercise the GLWB option (since, due to the step-ups at these ages, exercise is never optimal at intervening ages). We consider one specification in which the individual decides at age 65 whether to exercise the GLWB option at ages 65, 70, 75, or 80; and another in which at ages 65, 70, and 75, considering realized investment returns, the individual chooses between exercising the GLWB option and postponing the decision for five years.

Total consumption C_t equals the sum of Social Security benefits S_t, the GLWB payments G_t, any withdrawals N_t from the non-variable annuity account, and any withdrawals V_t from the variable annuity exceeding the GLWB payment. More specifically, the budget constraints without taxes are as follows:

$$C_t = S_t + G_t + N_t + V_t \quad (2)$$

$$F_{t+1} = (F_t - N_t)[(1 - \Theta_{Ft})(1 + r_s) + \Theta_{Ft}(1 + r_b) - \varphi_F] \quad (3)$$

$$A_{t+1} = max(0, (A_t - G_t - V_t)[(1 - \Theta_{At})(1 + r_s) + \Theta_{At}(1 + r_b) - \varphi_A] - \varphi_W W_t) \quad (4)$$

$$W_{t+1} = max\left(A_{t+1}, W_t\left(1 - \frac{V_t}{A_t}\right)\right) \quad (5)$$

Equation (3), (4), and (5) show the laws of motion for non-annuity asset F_t, the variable annuity account value A_t, and the high-water mark W_t of the VA at time t, where r_b is the return on bonds, r_s is the return on stocks, Θ_{At} is the share of annuity wealth allocated to bonds at time t, and Θ_{Ft} is the share of financial wealth allocated to bonds at time t. Turning to fees, φ_A is the annuity investment, mortality and expense, and administrative fee as a percentage of annuity wealth. Fees cease if the annuity has been exhausted. φ_F is the financial asset investment management fee as a percentage of non-VA asset, and φ_W is the annual GLWB fee as a percentage of the high-water mark.

To avoid modeling a process for inflation and the term structure of interest rates, we assume a fixed real interest rate on long-term bonds. Campbell and Viciera (2002) argue that, for a long-term investor, a portfolio of long-term bonds of appropriate durations are the true risk-free asset; although their market value will fluctuate in response to movements in interest rates, the portfolio enables the investor to lock in future consumption. However, long-term bonds may be considered a risky asset by households with uncertain consumption requirements or who otherwise value liquidity. In practice, our approach understates the riskiness of bonds because most households hold bonds paying a nominal return and fail to choose bond portfolios with durations that insure their consumption. This simplification is not too consequential as our model predicts that households should allocate relatively small shares of their financial assets to bonds, because Social Security benefits have investment characteristics similar to those of low-risk bonds.

3.2 BEQUESTS

We follow Lockwood (2012, 2018) and assume that bequests are a luxury good. As Lockwood (2018) emphasized, a bequest function that fits multiple facets of late-life saving and insurance behavior well involves a bequest motive that is largely satisfied from incidental bequests (that is, made without much effort to restrict consumption to increase one's bequest, and largely resulting from dying earlier than the age at which it would be optimal to exhaust all financial assets). With such a bequest function, one must be quite wealthy (in which case the marginal utility of one's own consumption is low) for the marginal utility of incremental planned bequests to exceed the marginal utility of own consump-

20. We avoid the added complexity of making the annuity purchase age a choice variable. Dushi and Webb (2004) show that it can be optimal to delay the purchase of an immediate annuity, so our calculations will understate the value of immediate annuities and possibly also variable annuities.

tion. In this formulation, the individual trades off the marginal utility that they will obtain from additional consumption and the marginal utility that the heir will obtain from their additional consumption. The strength of the bequest motive will depend on the following factors: 1) the extent to which the individual prioritizes her own well-being relative to her heir's, 2) the level of consumption the heir would enjoy in the absence of the bequest, and thus the heir's marginal utility, 3) the period of years over which the bequest will be consumed, because per period consumption will be lower and the marginal utility of consumption higher over a longer period, and 4) the assumptions regarding rate of return and the rate of time preference.

The extent to which the individual prioritizes their own well-being is governed by the coefficient in the following bequest utility function:

$$v(b) = \alpha \left(\sum_{i=1}^{i=\infty} \beta^{i-1}\right) \frac{\left(y_g + \frac{b}{\sum_{i=1}^{i=\infty} (1+r)^{-(i-1)}}\right)^{1-\sigma}}{1-\sigma} \quad (6)$$

The α coefficient interacts with the coefficient of risk aversion σ. When the value of α is 0.5, the household would target a consumption level for their heirs that was 70.7 percent of their own when the coefficient of risk aversion is two (so the household is less risk-averse) versus 87.0 percent when the coefficient is five (so the household is more risk-averse and prefers a smoother and less risky consumption stream). Lockwood (2012) assumes that the heir has a permanent income equal to the individual's Social Security benefits.[21]

The above utility function abstracts from the decisions the heir faces about how to invest and consume the bequest. To avoid complications associated with modeling stochastic returns and precautionary savings, we assume the bequest is invested in a bond yielding a risk-free three percent. The only other complication is that the timing of the bequest is uncertain, depending on the date of death of the testator, requiring us to undertake a separate discounting of the heir's utility for each possible date of death.

3.3 OTHER ASSUMPTIONS

When held to maturity, Treasury Inflation Protected Securities have been a risk-free asset, providing a guaranteed return of principal and a guaranteed income in the meantime, and besides in some sensitivity analysis, we assume no inflation surprises. We assume bonds yield a real return of 2.34%, the yield on 10-year constant maturity Treasury Inflation Protected Securities in October 2023, higher than rates following the Great Financial Crisis but lower than historic rates.[22] We disfavor using historic stock returns because both the equity premium and the risk-free rate have likely declined in recent years (Diamond 1999, Graham and Harvey 2015). Instead, we use an expected return equal to the inverse of the October 2023 forward price-earnings ratio on large capitalization stocks, giving an expected real return of 5.7%.[23]

Variable annuity characteristics vary considerably. We therefore rely on information about representative products while analyzing sensitivity to some important features. In our base case specification, we rely on the following market analysis conducted by Wade Pfau in March 2023 and relayed through personal communication.

21. We assume that when the heir dies, the bequest passes to the heirs in the same period. Assuming a dynasty with an infinite time horizon would not significantly change the result as it would have only a small effect on annual consumption and the marginal utility of that consumption.

22. Federal Reserve Bank of St. Louis. Market yield on U.S. Treasury Securities at 10-year constant maturity, as of 4 October 2023. https://fred.stlouisfed.org/series/DFII10

23. Yardeni Research Inc. Stock Market Briefing: Selected Pes. October 12, 2023. https://www.yardeni.com/pub/stockmktperatio.pdf

	Age of First Withdrawal	Guaranteed Payout Percentages [24]
	60-64	4.18%
	65-69	5.18%
	70-74	5.33%
	75-79	5.62%
	80+	5.7%
Annual Mortality and Expense and Administrative Charges	1.25%	
Underlying Account Fee for Investment Expenses	0.75%	
Annual Rider Fee	1.33%	
Rider Applies To	High-Watermark Benefit Base	

Variable Annuity Parameters

Our base case does not include a constraint on the variable annuity asset allocation, this being considered in our sensitivity analyses. Immediate annuity parameters are based on the average of prices observed on December 4, 2023, on the website of Blueprint Income.[25]

Assumptions related to registered index-linked annuities are based on the observation that insurers typically hedge those obligations by selling a call option and using the proceeds to purchase a put option. As of January 2024, the cost of a one-year put option on the S&P 500 at a strike price of 10% less than current market price approximately equals the proceeds of a one-year call at a strike price of 15.8% above the current price. Reflecting these option prices, we consider a floor of minus 10% percent, a cap of 15.8%, and 100% participation in S&P 500 returns, exclusive of a dividend yield of 1.6% (the dividend yield in October 2023). Registered index-linked annuities have no investment fees and we assume purchasers face the same GLWB fees as those applicable to traditional variable annuities.

4. RESULTS

Our model of optimal decision-making yields calculations of annuity equivalent wealth, the factor by which the wealth of someone aged 65 who is unable to purchase an annuity must be increased to make them just as well off (in expected utility terms) as the same individual with access to the annuity market. Strategies with higher expected utility yield higher annuity equivalent wealth.

We first present results for variable annuities, which have the simplest structure as an investment product, comparing them with immediate annuities as a benchmark for men and then for women. We consider optimal allocation behavior along with the impact of the bequest motive, inflation, and taxes. We then present results for registered index-linked annuities, which are growing in popularity.

24. The GLWB benefit is a percentage of the benefit base, that percentage being determined by reference to the age at which the GLWB option is exercised, as noted in this table. The percentage is applied to the current high-water mark of the contract value. As GLWB withdrawals reduce the contract value, exercise of the GLWB option reduces, but does not eliminate, the probability of hitting a new high-water mark. For typical contracts and for periods prior to exercise of the GLWB option, the high-water mark may be further increased by comparing the high-water mark of the contract value with the purchase price of the contract plus 7% simple interest (the "roll-up rate") and taking the higher of the two.
25. Blueprint income, a subsidiary of MassMutual, is an annuity marketplace that "offers a curated selection of the top 30 insurance companies" (Guaranteed Fixed & Income Annuities | Blueprint Income). The standard expected-utility model that we use here will result in lower valuations when incorporating other features of many immediate annuities, such as period-certain payouts. Therefore, to avoid putting a thumb on the scale against immediate annuities, we leave out such features.

4.1 VARIABLE ANNUITIES

We start by assuming that individuals with a bequest motive choose optimal asset allocations both within and outside the variable annuity. Table 1A reports, for men, the resulting annuity equivalent wealth in various scenarios. These include VA allocations in 10% increments from 0-100% as a share of total financial assets and GLWB exercise ages of 65, 70, 75, 80, and the optimal exercise age. The final column shows immediate annuity allocations in the same 10% increments. The top and bottom panels show all of this for coefficients of risk aversion of two and five. Table 1B reports corresponding results for women. As a reminder, we assume Social Security income of $26,000 a year and financial assets of $400,000.

For a coefficient of risk aversion of two (corresponding to a relatively low level of risk aversion), annuity equivalent wealth is positive for low annuitization shares and early exercise of the GLWB option, and when positive, reaches values of over $20,000 for the VA and less than $10,000 for the immediate annuity. Thus, annuities offer moderate value for those who are not very risk averse, when incorporating investment risk into the Lockwood model of bequest motives. Annuity equivalent wealth reaches a peak at $22,329 for a variable annuity allocation of 50% and at $8,229 for an immediate annuity allocation of only 30%. In other words, a man who has access to an annuity would have to be compensated by $22,329 if denied the capacity to purchase a variable annuity and $8,229 if denied the capacity to purchase an immediate annuity. The peak VA value occurs for a GLWB exercise age of 65, and since this delivers more than an immediate annuity purchased at the same age, it shows that guaranteed lifetime income is especially valuable when it offers access to market returns, combined with protection from extreme market outcomes.

At a coefficient of risk aversion of five (corresponding to a relatively high level of risk aversion), annuity equivalent wealth reaches a substantially greater peak, at over $50,000 for the VA and over $30,000 for the immediate annuity. It is noteworthy that the VA continues to dominate the IA in this case. Annuity equivalent wealth reaches a maximum of $57,048 when the variable annuity allocation is 50%, compared to $35,059 when the immediate annuity allocation is 40%.

At both levels of risk aversion, annuity equivalent wealth for the variable annuity is maximized at a GLWB claim age of 65, so it is not worth preserving the option to exercise the GLWB at other ages. Delayed exercise of the GLWB option may result in the fund hitting a new high-water mark and increases the percentage of the high-water mark used to calculate the income payments. The increases are small, though – smaller than the inflation-indexed increases for delayed claiming of Social Security – and insufficient to compensate the household for delay. When we consider higher age-related increases in GLWB rates, we find that, although it can be optimal to delay exercise of the GLWB option to older ages, there is little additional benefit to retaining the option versus pre-committing at age 65 to what appears from that vantage point to be the optimal age.

Results for women, in Table 1B, show higher annuity equivalent wealth for both annuity types, reflecting women's greater longevity and, moreover, unisex pricing of the variable annuity GLWB. The result is that the money's worth of immediate annuities (the expected present value of benefits as a percent of premiums) is currently about three percentage points higher for women than for men and this contributes to women's greater willingness to pay.[26] Variable annuity equivalent wealth for women who are not very risk averse reaches a peak of $31,993 (about half again as much as the peak for men), at an optimal allocation of 50% when risk aversion is low. When risk aversion is high, variable annuity equivalent wealth is maximized at $74,995, also at an allocation of 50%. Annuity equivalent wealth is similarly higher for the immediate annuity for women when risk aversion is high. Overall, these results reveal that VAs dominate immediate annuities across the board – for both women and men and for both relatively high and relatively low levels of risk aversion– when we incorporate investment risk into a model like that in Lockwood (2012), with longevity risk, a bequest motive, and realistic fees.

26. This result is somewhat sensitive to the discount rate used and it is unclear what is the most appropriate rate. At a higher discount rate (and, notably, Lockwood (2018) estimates a considerably higher discount rate), the money's worth gap narrows.

4.2 THE IMPACT OF THE BEQUEST MOTIVE

Now, we explore the bequest motives in the model. We begin by considering the impact of eliminating any bequest motive (Tables 2A for men and 2B for women). Eliminating utility from bequests increases the value of both the variable and immediate annuities and increases the optimal allocation to either. The immediate annuity now dominates the VA, since it provides more complete longevity insurance; this further demonstrates that the exclusive focus in Lockwood (2012) on immediate annuities overlooks part of the protection that real-world annuities offer. For less risk-averse men (with a coefficient of risk aversion of two), annuity equivalent wealth is $69,264 for the variable annuity and $91,060 for the immediate annuity, with optimal allocations of 70% and 80%, respectively. For the more risk-averse (with a coefficient of risk aversion of five), annuity equivalent wealth is quite high, at $161,583 for the variable annuity and $228,164 for the immediate annuity, with optimal allocations of 70% and 80%, respectively. In each case, the optimal GLWB exercise age remains 65. Although the less risk averse might be expected to prefer greater exposure to the equity premium, they nonetheless allocate most of their financial assets to the immediate annuity. They increase this further when the variable annuity is available, offering the equity premium plus both partial protection against poor investment returns, and longevity insurance. In the real world, few households allocate almost all their financial assets to variable annuities, and we recognize that our model abstracts from real world considerations.[27]

Results for women are similar and have analogous patterns as those reported for Tables 1A and 1B. Annuity equivalent wealth for the variable annuity is higher than for men, reflecting unisex pricing, and is higher for the immediate annuity reflecting the higher female immediate annuity money's worths that we observe in the data.

We also consider the impact of annuity type on expected bequests, reverting to the model with a bequest motive. We focus on women in our analysis going forward, since annuity pricing is more favorable to them and since they often outlive their husbands. The results are shown in Table 3. We calculate the expected bequest by undertaking 10,000 simulations of optimal behavior with stochastic mortality and investment returns. For each simulation, we first discount the bequest back to age 65 (using our assumed time discount rate and inflation rate) and then calculate the average across the simulations. Annuitization of either type reduces the share of wealth that is expected to be passed as a bequest. Consider an individual with a coefficient of risk aversion of two (who is not very risk averse) and, as before, initial wealth of $400,000. When the individual allocates the optimal share of wealth to a VA, the expected bequest is $165,442, and when the individual allocates the optimal share of wealth to an immediate annuity, it is $163,308 -- which are quite similar, although the VA offers greater value by virtue of offering a higher stream of consumption. This demonstrates the point in Lockwood (2012) that bequests are largely incidental for most individuals. The corresponding amounts percentages for a more risk averse individual are $153,178 for a VA and 151,616 for an immediate annuity.[28]

4.3 THE IMPACT OF INFLATION

The GLWB guarantees are expressed in nominal terms and are less valuable in real terms at higher inflation rates. Table 4 reports annuity equivalent wealth at inflation rates of 0% or 4%, compared to 2% which we assumed in our baseline.[29] In each case, we hold real stock and bond returns constant, and we assume that inflation is perfectly anticipated.

The assumed inflation rate has a substantial effect on both annuity equivalent wealth and the optimal alloca-

27. By way of comparison, participants in defined benefit pension plans (albeit a relatively small group of retirees at this point) often hold few financial assets outside their plans. This mimics the results we find, without questions being raised as to the suitability of their asset allocation.
28. With only a single risk-free asset, we would expect the higher intertemporal elasticity of substitution of the less risk-averse individual to result in a faster drawdown of assets and a smaller bequest. This is offset, however, by the larger asset allocation to higher expected return equities.
29. Households can protect themselves against anticipated inflation by purchasing an immediate annuity with an income that increases at a pre-determined rate. These tend to have lower money's worths to households with population average mortality because a larger share of payments are made at the older ages to which annuitants are disproportionately likely to survive. We do not report results for these products, reflecting our primary focus on variable and fixed indexed annuities.

tion to the variable annuity, but no effect on the optimal GLWB exercise age. Annuity equivalent wealth is significantly lower at higher assumed inflation rates. Assuming an optimal allocation to the variable annuity, a bequest motive, and an optimal GLWB exercise age, when risk aversion is high, female annuity equivalent wealth is $45,430 at a 0% inflation rate, the $31,993 reported above at 2% inflation, and $19,917 at 4% inflation, reflecting the reduced value of the nominal GLWB guarantee at higher assumed inflation rates. The assumed inflation rate has little effect on the optimal variable annuity allocation. However, the assumed inflation rate has a strong effect on the value of the immediate annuity and the ranking of the annuity types. The mechanism is that the variable annuity returns are assumed to be real – the assumed stock return, although risky, is expressed in real terms, as is the bond return, whereas the immediate annuity return is expressed in nominal terms. A permanent increase in the inflation rate would likely raise nominal interest rates commensurately. If individuals purchased annuities with increasing benefits, the equivalent wealth of immediate annuities would be invariant to the inflation rate.[30]

The question then arises – would individuals prefer a less generous real guarantee to a more generous nominal guarantee? Relative to nominal annuities, inflation-indexed annuities make larger payments at older ages, ages at which annuities are a highly cost-effective means of financing consumption, relative to a drawdown of unannuitized wealth. Moreover, individuals will likely be willing to pay an additional premium for the elimination of uncertainty as to the real value of their annuity.

Although inflation-indexed immediate annuities no longer exist in the U.S., it is relatively straightforward to estimate a lower bound on the prices at which they might be sold, using the term structure of interest rates of Treasury Inflation Protected Securities and annuitant mortality tables.[31] Estimating prices for inflation-indexed GLWBs is considerably more complex because the insurer needs to hedge both investment and inflation risk. We defer pricing of inflation-indexed GLWBs and calculating willingness-to-pay to future research.

4.4 THE IMPACT OF ASSET ALLOCATION

50:50 EQUITY/BOND ALLOCATION

Relative to an optimal asset allocation both inside and outside the GLWB, adopting a 50/50 equity/bond asset allocation (which individuals may choose by following a rule-of-thumb) substantially reduces annuity equivalent wealth even for relatively risk averse individuals, from $74,995 (Table 5, lower panel, column one, 50% allocation of wealth to variable annuity) to $61,091 (Table 5, lower panel, column four, 50% allocation to variable annuity). To put this in context, this is equivalent to more than 100 basis points of fees, which reduces annuity equivalent wealth by $13,904. Corresponding reductions for less risk averse individuals are $31,993 (column one, upper panel, 50% allocation to variable annuity) to $22,182 (column four, upper panel, 50% allocation).

CHOOSING THE SAME ALLOCATION FOR BOTH VA AND NON-VA ASSETS

In contrast, adopting a similarly naïve strategy of choosing the same investment allocation in both accounts has only a small effect on annuity equivalent wealth. For relatively risk averse individuals, annuity equivalent wealth drops from $74,995 to $68,658 (Table 5, lower panel, columns one and three). For relatively less risk averse individuals, the optimal allocation in both accounts is almost 100% to stocks, so the constraint has virtually no effect (Table 5, upper panel, columns one and three), reducing annuity equivalent wealth $31,993 to $31,923.

30. We do not have the computing capacity at this time to model inflation shocks, which involves making difficult assumptions about inflation expectations. Therefore, our analysis offers a baseline in which inflation is perfectly forecasted by investors and insurers.

31. This pricing approach assumes that insurers experience the same degree of adverse selection in both nominal and inflation-indexed annuities. Yet, Finkelstein and Poterba (2004) established that inflation-indexed annuities suffer from greater adverse selection. Furthermore, insurers typically invest in higher-return corporate bonds, and it would be more costly to hedge inflation risk than is implied by the yield difference between fixed and index-linked Treasury bonds.

CAPPING THE VARIABLE ANNUITY STOCK ALLOCATION AT 70%

An added complication is that insurers may place limits on the riskiness of a variable annuity portfolio. When choosing a variable annuity, individuals may need to trade-off higher fees against the benefit of a closer-to-optimal asset allocations in both the variable annuity and rest of the portfolio.

Because purchasing a VA with a GLWB option might induce excessive risk-taking, from the perspective of the insurer, we consider the impact if VA sellers capped the equity allocation at 70%. The impact is to significantly reduce annuity equivalent wealth. When the individual has a strong bequest motive and risk aversion is relatively low (with a coefficient of risk aversion of two), annuity equivalent wealth is now maximized at $22,383, at a 50% variable annuity share of financial assets. When risk aversion is relatively high, at the unchanged optimal VA share of financial assets, annuity equivalent wealth declines by $8,842.

4.5 THE IMPACT OF FEES AND TAXES

In Table 6, we consider, the impact of reducing fees below our baseline. We find that a 100-basis point (bp) reduction in fees increases the valuation of VAs for either level of risk aversion by about $12,000, when the VA share is chosen optimally. It does not change the optimal share of wealth allocated to a VA, and the increase in valuations for the VA exceeds the increase for immediate annuities.

When held within a tax-deferred account such as an Individual Retirement Account or 401(k), the tax treatment of variable annuities is identical to that of stocks and bonds held directly or through a mutual or exchange traded fund. Contributions are deductible, investment returns are free of tax, and withdrawals are taxed as income. In the case of Roth accounts, contributions are not tax deductible but withdrawals are not taxed.

Otherwise, the tax treatments of annuities and of stocks and bonds held outside annuities differ. For non-annuity assets, interest on bonds and short-term capital gains are taxed at normal income tax rates, while dividends and long-term capital gains on stocks are taxed at lower rates. Tax on annuity investment returns is deferred until the annuity is liquidated, when the excess of proceeds over purchase price is taxed as income; or until an income option is exercised, when the portion of income that represents the return of capital is exempt from tax. Annuities are thus an effective means of deferring income from periods when the household faces high marginal tax rates to periods after retirement, when tax rates may be lower – a time period that we are not considering, since we focus on the decision to purchase an annuity when the individual is retired at age 65. However, annuity investors forego the benefit of the lower marginal tax rate on dividends and long-term capital gains.   Absent tax considerations, households may want to maximize the value of the GLWB put option by holding their riskiest assets (stocks) in their variable annuity, but tax considerations may dictate otherwise.

The share of Social Security benefits subject to tax depends on the household's "combined income," and the phase out of the Social Security tax exemption can result in high marginal tax rates (Reichenstein and Meier, 2018). However, most retired households with assets in the ranges we consider hold much of their wealth in tax-deferred accounts. So, tax considerations weigh heavily only for the subset of the wealthy minority who hold significant wealth outside tax-deferred accounts.

To evaluate the role of taxes, we now consider a household with $1 million in financial assets, a level at which tax liabilities may matter more. We assume that this amount is held in tax deferred accounts. At 2023 tax rates, $50,000 drawn from such an account, together with Social Security benefits of $26,000, would result in an average tax rate of just over 10%. We defer to future consideration the benefits of variable annuities in shifting income from before to after retirement for households with lower post-retirement tax rates. In Tables 7A (low risk aversion, with a coefficient of two) and 7B (high risk aversion, with a coefficient of five), taxes reduce consumption possibilities late in life and thus the value of annuitization, but also imply that larger pre-tax annuity-equivalent wealth is required to yield the same post-tax benefit. The two effects almost exactly offset each other, and taxes have only a small effect on annuity equivalent wealth.

4.6 REGISTERED INDEX-LINKED ANNUITIES

Given the growing popularity of registered index-linked annuities, which allow individuals to choose pre-set floors and/or ceilings for returns, we consider how they affect optimal choices. Following Campbell and Viciera (2002), our model treats the long-term bond as a risk-free asset. Assuming a mean and standard deviation of equity returns of 5.7% and 18% respectively, inflation of 2%, and a dividend yield of 1.6%, the nominal cap and floor of 15.8% and minus -10% yields an expected real return on a RILA of 0.26%, less than the assumed 2.34% real return on the long-term bond. This “equity discount” offsets the value of the GLWB and makes the RILA less attractive to those who are not very risk averse.

Tables 8 and 9 shows results for a RILA with a buffer and for a fixed RILA. For individuals who are quite risk averse (with a coefficient of five), annuity equivalent wealth is maximized at $75,212 a RILA share of 50% (Table 8, when it has a buffer), while in contrast annuity equivalent wealth is $27,052 when risk aversion is low. For a fixed RILA, annuity equivalent wealth is approximately $12,000-16,000 lower at either level of risk aversion, but it is still higher than that of an immediate annuity. Households prefer the lower degree of protection against downside risk and greater upside potential of the RILA with a buffer.

Although we would likely obtain different results in a model in which the long-term bond were a risky asset, we note that our assumed equity premium is sufficiently large to crowd out bonds, even under the assumption of a fixed long-term interest rate.

4.7 IMPLICATIONS FOR HOUSEHOLDS AND THEIR ADVISORS

Theoretical models indicate that all except the most risk-averse households should retain exposure to the equity market, even in retirement. While immediate annuities provide valuable insurance against outliving one’s wealth, households that purchase an immediate annuity forego exposure to equities. In theory, a variable immediate annuity can offer both longevity insurance and exposure to the equity market. However, this product is not currently sold in the U.S. market. Thus, immediate annuities can be viewed, at best, as substituting for the bond portion of the household’s portfolio.

On the other hand, households investing in equities and bonds must accept the possibility of very low consumption later in retirement should markets perform poorly. Bond investors can hedge interest rate risk by holding bonds of appropriate durations but still face default risk. As an alternative, variable annuities with a lifetime income option provide exposure to the equity premium, plus a floor below which the household’s income can never fall, plus liquidity, plus the possibility of passing on a bequest -- though at the cost of a lower income guarantee than that obtainable from an immediate annuity. Our modeling shows that annuities improve household financial well-being along all those dimensions, while the choice among annuity types depends on preferences – what advisors refer to as a household’s ability and willingness to bear risk – and beliefs about future returns and inflation. Moreover, to maximize the benefit of variable annuities, households must make appropriate decisions throughout the life of the annuity.

At the time of purchase, households must consider fees and benefits. Unlike immediate annuities, variable annuities are not a uniform product. Higher fees may purchase more valuable benefits – for example an ability to invest in riskier assets, a higher annuity rate as a percentage of the high-water mark, or a larger age-related increase in the annuity rate. All this points to the need for households to receive appropriate and continuing professional advice.

5. CONCLUSIONS

Immediate annuities offer full insurance against both investment and longevity risk. The price of this insurance is foregoing the higher expected returns on equities and the possibility of passing on unspent wealth as a bequest. Variable annuities with a lifetime income option offer a compromise. The income floor is lower than that obtainable from an immediate annuity, but the household retains the possibility of passing on a bequest and increasing its income should financial markets perform well, while obtaining insurance against the confluence of two bad financial outcomes: living unusually long and experiencing poor investment returns. Registered index-linked annuities perform a similar function.

We use the models developed to explain demand for immediate annuities (which in many cases have amplified the annuity puzzle instead) to analyze demand for real-world variable annuities. Our model incorporates numerous critical decisions: whether to purchase an immediate or variable annuity with a GLWB rider; how to allocate assets inside and outside of a variable annuity; when to convert annuity assets into lifetime income; and how much to bequeath to heirs. Our results establish that at least partial annuitization, whether through an immediate annuity or a variable annuity with a GLWB rider, dominates no annuitization, even when bequest motives are relatively strong. Moreover, our model reveals that variable annuities dominate not only no annuitization but also immediate annuities in the circumstances that we consider.

Beyond that, the ranking of variable and immediate annuities depends on household beliefs and preferences, as well as on the costs associated with the various products – factors that financial advisors can observe about individuals with better precision than we can. For some households, allocating part of age-65 financial assets to immediate annuities, whereas for others, allocating part to variable annuities, makes sense. Therefore, we cannot say with certainty that there is an annuity puzzle waiting to be solved.

We acknowledge several limitations of our study. Our model does not incorporate the potential benefit of tax deferral when annuities are purchased pre-retirement. Our model does not consider long-term care cost risk, nor the ability of annuities without a surrender value to preserve income for a surviving spouse. These issues are complex and we defer investigation to future research.

AUTHORS

Leora Friedberg is Associate Professor of Economics and Public Policy at the University of Virginia.

Guoxuan Ma is Assistant Professor of Economics in the Research Institute for Global Value Chains at the University of International Business and Economics

Wei Sun is an Associate Professor at China Financial Policy Research Center and School of Finance, Renmin University of China.

Anthony Webb is Senior Fellow at the Schwartz Center for Economic Policy Analysis, The New School.

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REFERENCES

Blanchett, David. 2023. "Protected Lifetime Income Benefits: The Next Generation GLWB." Retirement Income Institute Original Research #021-2023, June. https://www.protectedincome.org/wp-content/uploads/2023/06/RP-21_Blanchett_v4.pdf .

Brown, Jeffrey R. and James Poterba 2000. "Joint Life Annuities and Annuity Demand by Married Couples." *Journal of Risk and Insurance,* 67(4): 527-53.

Brown Jeffrey R., Olivia S. Mitchell, and James M. Poterba. 2001. "The Role of Real Annuities and Indexed Bonds in an Individual Accounts Retirement Program," in *Risk Aspects of Investment-Based Social Security Reform,* edited by J. Campbell and M. Feldstein.

Chetty, Raj. 2006 "A New Method of Estimating Risk Aversion." *American Economic Review,* 96 (5): 1821-1834.

Chen, Anqi and Alicia H. Munnell. 2020. "How Much Taxes Will Retirees Owe on Their Retirement Income?" Working Paper 2020-16. Chestnut Hill, MA: Center for Retirement Research at Boston College.

Davidoff, Thomas, Brown, Jeffrey R., and Diamond, Peter A. 2005. "Annuities and individual welfare." *American Economic Review* 95 (5), 1573–1590.

De Nardi, Mariacristina, Eric French, and John B. Jones. 2010. Why Do the Elderly Save? The Roel of Medical Expenses. *Journal of Political Economy* Vol. 188 No. 1 February. Pages 39-75.

Diamond, Peter A. What Stock Market Returns to Expect for the Future." Center for Retirement Research Issue in Brief #2.

Dushi, Irena, and Anthony Webb. 2004. "Household Annuitization Decisions: Simulations and Empirical Analyses." *Journal of Pension Economics & Finance,* 3(2), 109-143.

Dushi, Irena and Anthony Webb. 2006. Rethinking the Sources of Adverse Selection in the Annuity Market in *Competitive Failures in Insurance Markets: Theory and Policy Implications*. Cambridge MA. MIT Press. Pages 185-212. Eds. Chiappori, Pierre Andre and Christian Gollier.

Finkelstein, Amy, and James Poterba. 2004. "Adverse Selection In Insurance Markets: Policyholder Evidence From The U.K. Annuity Market." *Journal of Political Economy* 112(1), pp. 183-208.

Friedberg, Leora and Anthony Webb. 2022. A Primer on Annuities: What Economists and Financial Professionals Need to Know about Variable Annuities, Fixed Indexed Annuities, and Registered Index-Linked Annuities. Retirement Income Institute Original Research #008-2022, July. https://www.protectedincome.org/wp-content/uploads/2022/07/RP-08-Friedberg-Webb_v5.pdf.

Gong, Guan and Anthony Webb. 2010. "Evaluating the Advanced Life Deferred Annuity: An Annuity People Might Actually Buy" *Insurance: Mathematics and Economics* Vol. 46 pages 210-221.

Graham, John R., and Campbell R. Harvey. 2015. "The Equity Risk Premium in 2015." Manuscript, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2611793.

Horneff, Vanya, Raimond Maurer, Olivia S. Mitchell, and Ralph Rogalla. 2015. "Optimal Life Cycle Portfolio Choice with Variable Annuities Offering Liquidity and Investment Downside Protection." *Insurance: Mathematics and Economics* 63 (July): 91–107.

Huang, Huaxiong, Moshe A. Milevsky, and Thomas S. Salisbury. 2014. "Optimal Initiation of a GLWB in a Variable Annuity: No Arbitrage Approach." *Insurance: Mathematics and Economics* 56 (May): 102–11.

Lockwood, Lee. 2012. "Bequest Motives and the Annuity Puzzle." *Review of Economic Dynamics*. Volume 15 No. 2. Pages 226–243.

Lockwood, Lee. 2018. "Incidental Bequests and the Choice to Self-Insure Late-Life Risks." *American Economic Review* Volume 108, No. 9. Pages 2513–2550.

Milevsky, Moshe A. and Thomas S. Salisbury. 2006. "Financial Valuation of Guaranteed Minimum Withdrawal Benefits." *Insurance: Mathematics and Economics* Vol. 38. Iss. 1. February Pages 21-38.

Mitchell, Olivia, James Poterba, Mark J. Warshawsky and Jeffrey R. Brown. 1999. "New Evidence on the Money's Worth of Individual Annuities" *American Economic Review,* Vol. 89 No. 5.

Moenig, Thomas and Daniel Bauer. 2016. Revisiting the Risk-Neutral Approach to Optimal Policyholder Behavior: A Study of Withdrawal Guarantees in Variable Annuities. *Review of Finance,* Volume 20, Issue 2, March 2016, Pages 759–794,

Moenig, Thorsten. 2021. Variable Annuities: Market Incompleteness and Policyholder Behavior. *Insurance: Mathematics and Economics* Vol. 99. July. Pages 63-78.

Moenig, Thorsten. 2021. It's RILA time: An Introduction to Registered Index-Linked Annuities. *Journal of Risk and Insurance*. Vol. 89 Issue 2. June. Pages 339-369.

Pashchenko, Svetlana. 2013. "Accounting for Non-Annuitization." *Journal of Public Economics* 98(C), 53-67.

Pfau, Wade. 2019. Safety-First Retirement Planning: An Integrated Approach for a Worry-Free Retirement. Self-Published. ISBN 978-1945640063

Reichenstein, William and William Meier. 2018. Understanding the Tax Torpedo and Its Implications for Various Retirees. *Journal of Financial Planning*, July

Rohwedder, Susann, Michael Hurd, and Peter Hudomiet. 2022. Explanations for the Decline in Spending at Older Ages. National Bureau of Economic Research Working Paper No. 30,460.

Steinorth, Petra, and Olivia S. Mitchell. 2015. "Valuing Variable Annuities with Guaranteed Minimum Lifetime Withdrawal Benefits." *Insurance: Mathematics and Economics* 64, 246–258.

Sun, Wei, and Anthony Webb. 2011. "How Much do Households Really Lose By Claiming Social Security At Age 62." *Journal of Risk and Insurance* 78(4), pp. 907-992.

Webb, Anthony. 2021a. "What Lessons, If Any, Can Practitioners Apply From Academic Economists about the Role of Annuities in Post-Retirement Asset Decumulation?" Retirement Income Institute #003-2021, February. https://www.protectedincome.org/wp-content/uploads/2021/08/RS981_LR-03-Webb-FN-AH-Feb3.pdf.

Webb, Anthony. 2021b. "What Can Scholarly Research Tell Us about Cognitive and Behavioral Impediments to Annuitization? Do These Impediments Vary with Race/Ethnicity and Gender?" Retirement Income Institute #006-2021, November. https://www.protectedincome.org/wp-content/uploads/2021/12/E-06-Webb-FN-AH_PROOFED_8.25.21_Nov-2021.pdf .

Xiong, James X., Roger G. Ibbotson, Thomas M. Idzorek, and Peng Chen. 2010. "The Equal Importance of Asset Allocation and Active Management." *Financial Analysts Journal* 66 (2): 22–30.

Xiong, James X., Thomas Idzorek, and Peng Chen. 2010. "Allocation to Deferred Variable Annuities with GMWB for Life." *Journal of Financial Planning* 23, 42-50.

	GLWB EXERCISE AGE					
CRRA = 2 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	7,679	5,588	2,002	-2,431	7,679	4,778
0.2	14,397	9,616	1,845	-7,562	14,397	7,837
0.3	17,872	10,403	-2,372	-17,328	17,872	8,229
0.4	21,025	9,185	-10,613	-31,249	21,025	6,816
0.5	22,329	5,193	-23,003	-47,686	22,329	3,982
0.6	20,642	-5,834	-44,030	-66,426	20,642	-3,168
0.7	16,771	-23,093	-66,738	-78,339	16,771	-17,101
0.8	5,783	-51,937	-77,382	-85,467	5,783	-40,403
0.9	-14,596	-71,187	-82,364	-91,088	-14,596	-81,590
1	-64,215	-73,531	-85,304	-94,382	-64,215	-150,135
	GLWB EXERCISE AGE					
CRRA = 5 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	17,854	16,411	12,432	6,607	17,854	16,750
0.2	31,512	27,305	18,112	5,417	31,512	27,326
0.3	42,897	32,041	15,359	-5,115	42,897	32,871
0.4	52,077	31,643	4,032	-24,101	52,077	35,059
0.5	57,048	25,718	-14,724	-49,152	57,048	32,801
0.6	51,547	5,074	-51,193	-79,837	51,547	20,141
0.7	37,338	-29,329	-90,066	-104,370	37,338	-10,672
0.8	10,648	-82,898	-127,596	-119,383	10,648	-68,243
0.9	-46,231	-147,023	-141,330	-129,509	-46,231	-147,129
1	-157,738	-152,527	-145,288	-136,009	-130,072	-263,687

TABLE 1A. *Annuity equivalent wealth, male with bequest motive*

Source: Authors' calculations. Household is assumed to have $26,000 Social Security income and $400,000 financial assets. CRRA refers to the coefficient of relative risk aversion, with 5 being more risk averse than 2. VA share refers to the share of financial assets invested in a variable annuity. Annuity equivalent wealth is the factor by which the wealth of someone aged 65 who is unable to purchase an annuity must be increased to make them just as well off (in expected utility terms) as the same individual with access to the annuity market. The bequest motive is parameterized as in Lockwood (2012). An inflation rate of 2% is assumed.

	GLWB EXERCISE AGE					
CRRA = 2 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	10,088	8,067	4,081	-1,309	10,088	7,200
0.2	18,978	14,174	5,507	-5,986	18,978	12,439
0.3	24,264	16,714	2,362	-16,007	24,264	14,959
0.4	29,204	17,046	-5,319	-30,960	29,204	15,787
0.5	31,993	14,316	-17,859	-49,132	31,993	15,232
0.6	31,913	4,204	-39,942	-69,107	31,913	10,858
0.7	29,597	-12,709	-64,959	-81,771	29,597	965
0.8	19,762	-42,539	-76,429	-88,833	19,762	-16,865
0.9	565	-65,139	-81,158	-94,245	565	-55,577
1	-52,141	-67,295	-83,965	-97,420	-52,141	-124,238
	GLWB EXERCISE AGE					
CRRA = 5 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	21,237	20,017	15,738	8,807	21,237	19,872
0.2	40,132	33,968	23,662	8,467	40,132	33,668
0.3	55,593	44,541	22,198	-2,578	55,593	46,230
0.4	67,656	46,643	11,203	-23,396	67,656	53,162
0.5	74,995	39,496	-8,492	-51,646	74,995	55,791
0.6	72,368	17,514	-47,339	-85,023	72,368	45,119
0.7	61,392	-17,046	-90,683	-113,559	61,392	19,373
0.8	28,456	-76,487	-133,582	-130,421	28,456	-25,549
0.9	-26,590	-146,403	-146,006	-140,121	-26,590	-106,652
1	-154,158	-152,449	-149,613	-144,756	-136,355	-240,108

TABLE 1B. *Annuity equivalent wealth, female with bequest motive*

Source: Authors' calculations. Notes: See Table 1A.

	GLWB EXERCISE AGE					
CRRA = 2 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	17,425	15,861	11,004	2,742	17,425	20,895
0.2	31,717	27,237	16,257	-26	31,717	39,042
0.3	43,017	33,471	15,980	-7,080	43,017	55,353
0.4	53,841	38,443	12,941	-15,518	53,841	68,361
0.5	61,318	40,225	8,159	-23,551	61,318	79,462
0.6	65,808	37,152	-81	-33,384	65,808	86,025
0.7	69,264	33,564	-7,430	-41,523	69,264	88,416
0.8	66,664	29,187	-13,297	-47,523	66,664	91,060
0.9	67,908	26,609	-18,184	-53,158	67,908	84,958
1	66,917	24,098	-21,607	-56,765	66,917	79,928
	GLWB EXERCISE AGE					
CRRA = 5 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	39,638	40,532	34,196	23,726	41,250	47,834
0.2	81,746	79,627	64,255	32,075	82,187	94,625
0.3	108,560	103,044	77,340	30,412	108,560	127,311
0.4	129,896	118,326	80,219	23,305	129,896	151,482
0.5	145,992	128,184	77,702	14,362	145,992	180,712
0.6	154,784	125,801	60,786	-2,057	154,784	203,165
0.7	161,583	120,289	41,674	-17,459	161,583	213,567
0.8	159,045	107,236	25,703	-31,484	159,045	228,164
0.9	156,987	97,337	15,644	-44,334	156,987	216,472
1	151,725	89,283	7,895	-53,849	151,725	204,940

TABLE 2A. *Annuity equivalent wealth, male without bequest motive*

Source: Authors' calculations. Household is assumed to have $26,000 Social Security income and $400,000 financial assets. CRRA refers to the coefficient of relative risk aversion, with 5 being more risk averse than 2. VA share refers to the share of financial assets invested in a variable annuity. Annuity equivalent wealth is the factor by which the wealth of someone aged 65 who is unable to purchase an annuity must be increased to make them just as well off (in expected utility terms) as the same individual with access to the annuity market. An inflation rate of 2% is assumed.

	GLWB EXERCISE AGE					
CRRA = 2 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	19,086	17,770	13,187	4,865	19,089	20,068
0.2	35,380	30,891	20,503	3,706	35,380	37,159
0.3	48,838	40,300	22,085	-2,433	48,838	52,728
0.4	61,340	47,392	20,576	-10,631	61,340	65,119
0.5	70,373	51,185	16,844	-19,028	70,373	75,669
0.6	76,452	49,701	8,946	-29,460	76,452	81,767
0.7	81,435	47,337	1,482	-38,146	81,435	83,797
0.8	80,213	41,964	-4,821	-44,729	80,213	86,052
0.9	82,476	38,768	-9,785	-50,612	82,476	79,785
1	81,778	35,641	-13,216	-54,338	81,778	74,127
	GLWB EXERCISE AGE					
CRRA = 5 VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	41,939	43,308	37,526	26,048	43,954	45,276
0.2	85,786	84,339	70,043	37,964	86,553	90,777
0.3	113,940	109,437	85,295	37,675	113,941	122,813
0.4	136,438	126,173	90,317	29,599	136,438	146,481
0.5	153,503	137,460	89,428	21,051	153,503	171,203
0.6	165,621	136,794	74,087	4,740	165,621	192,677
0.7	177,894	132,649	55,695	-11,065	177,894	202,291
0.8	176,781	120,403	34,765	-25,608	176,781	215,889
0.9	175,146	110,652	24,026	-38,577	175,146	203,363
1	166,531	102,821	16,514	-48,166	166,531	187,573

TABLE 2B. *Annuity equivalent wealth, female without bequest motive*

Source: Authors' calculations. Notes: See Table 2A.

	CRRA = 2		CRRA = 5	
Annuity share	Variable Annuity	Immediate Annuity	Variable Annuity	Immediate Annuity
0	189,732	189,732	172,317	172,317
0.1	182,716	180,214	164,040	164,116
0.2	178,120	176,014	158,012	159,864
0.3	172,452	171,038	155,637	161,857
0.4	168,115	163,308	153,953	155,148
0.5	165,442	157,817	153,178	151,616
0.6	160,442	150,308	151,362	141,721
0.7	154,056	131,430	144,486	120,413
0.8	140,394	98,886	126,057	89,576
0.9	107,154	54,027	97,170	50,134
1	55,562	0	93,638	0

TABLE 3. *Expected bequest as values of initial wealth, female with bequest motive*

Source: Authors' calculations. Household is assumed to have $26,000 Social Security income and $400,000 financial assets. CRRA refers to the coefficient of relative risk aversion, with 5 being more risk averse than 2. Annuity share refers to the share of financial assets invested in an annuity. The bequest motive is parameterized as in Lockwood (2012). An inflation rate of 2% is assumed.

CRRA = 2	INFLATION = 2%		INFLATION = 0%		INFLATION = 4%	
VA share	VA	Immediate annuity	VA	Immediate annuity	VA	Immediate annuity
0	0	0	0	0	0	0
0.1	10,088	7,200	13,069	12,982	7,924	1,868
0.2	18,978	12,439	24,618	23,955	14,297	1,966
0.3	24,264	14,959	33,187	32,425	17,258	-1,064
0.4	29,204	15,787	41,179	38,145	19,518	-5,088
0.5	31,993	15,232	45,430	39,346	19,917	-10,582
0.6	31,913	10,858	44,358	31,159	17,184	-19,083
0.7	29,597	965	34,232	13,946	13,487	-31,108
0.8	19,762	-16,865	13,830	-12,829	4,273	-46,054
0.9	565	-55,577	-18,030	-59,096	-9,150	-75,329
1	-52,141	-124,238	-74,033	-122,405	-43,950	-140,034
CRRA = 5	INFLATION = 2%		INFLATION = 0%		INFLATION = 4%	
VA share	VA	Immediate annuity	VA	Immediate annuity	VA	Immediate annuity
0	0	0	0	0	0	0
0.1	21,237	19,872	23,042	24,758	20,166	13,433
0.2	40,132	33,668	46,774	50,519	34,005	22,155
0.3	55,593	46,230	63,278	67,012	45,704	25,167
0.4	67,656	53,162	73,341	72,611	54,955	25,802
0.5	74,995	55,791	74,750	66,154	60,184	24,375
0.6	72,368	45,119	61,304	36,657	55,633	17,176
0.7	61,392	19,373	27,250	-6,531	47,342	1,510
0.8	28,456	-25,549	-18,737	-69,682	28,640	-27,826
0.9	-26,590	-106,652	-89,307	-149,045	-2,574	-94,367
1	-136,355	-240,108	-166,928	-253,022	-110,815	-234,494

TABLE 4. *Impact of inflation on annuity equivalent wealth, female with bequest motive*

Source: Authors' calculations. Note: Authors' calculations. Household is assumed to have $26,000 Social Security income and $400,000 financial assets. CRRA refers to the coefficient of relative risk aversion, with 5 being more risk averse than 2. VA share refers to the share of financial assets invested in a variable annuity. Annuity equivalent wealth is the factor by which the wealth of someone aged 65 who is unable to purchase an annuity must be increased to make them just as well off (in expected utility terms) as the same individual with access to the annuity market. The bequest motive is parameterized as in Lockwood (2012).

CRRA = 2	VARIABLE ANNUITY				
VA share	Base case	Cap at 70%	Same allocation VA and non-VA	50/50	Immediate annuity
0	0	0	0	0	0
0.1	10,088	7,833	10,048	7,214	7,200
0.2	18,978	14,777	18,922	13,866	12,439
0.3	24,264	18,016	24,201	17,060	14,959
0.4	29,204	21,340	29,138	20,691	15,787
0.5	31,993	22,383	31,923	22,182	15,232
0.6	31,913	21,028	31,837	21,897	10,858
0.7	29,597	17,586	29,512	19,884	965
0.8	19,762	6,228	19,673	10,196	-16,865
0.9	565	-13,944	492	-6,773	-55,577
1	-52,141	-68,370	-52,141	-48,340	-124,238
CRRA = 5	VARIABLE ANNUITY				
VA share	Base case	Cap at 70%	Same allocation VA and non-VA	50/50	Immediate annuity
0	0	0	0	0	0
0.1	21,237	19,152	19,249	17,915	19,872
0.2	40,132	34,804	35,326	31,873	33,668
0.3	55,593	48,383	49,556	43,833	46,230
0.4	67,656	59,612	61,466	54,711	53,162
0.5	74,995	66,153	68,658	61,091	55,791
0.6	72,368	63,629	66,671	58,783	45,119
0.7	61,392	52,915	56,204	48,128	19,373
0.8	28,456	23,300	25,843	19,211	-25,549
0.9	-26,590	-31,988	-28,623	-39,358	-106,652
1	-136,355	-142,503	-136,355	-138,986	-240,108

TABLE 5. *Impact of asset allocation on annuity equivalent wealth, female with bequest motive*

Source: Authors' calculations. Household is assumed to have $26,000 Social Security income and $400,000 financial assets. CRRA refers to the coefficient of relative risk aversion, with 5 being more risk averse than 2. VA share refers to the share of financial assets invested in a variable annuity. Annuity equivalent wealth is the factor by which the wealth of someone aged 65 who is unable to purchase an annuity must be increased to make them just as well off (in expected utility terms) as the same individual with access to the annuity market. The bequest motive is parameterized as in Lockwood (2012). An inflation rate of 2% is assumed.

CRRA = 2	VARIABLE ANNUITY			
VA share	Base case	-50 pts	-100 pts	Immediate annuity
0	0	0	0	0
0.1	10,088	11,178	12,359	7,200
0.2	18,978	21,013	23,208	12,439
0.3	24,264	27,231	30,411	14,959
0.4	29,204	32,933	37,862	15,787
0.5	31,993	37,337	43,325	15,232
0.6	31,913	37,996	44,763	10,858
0.7	29,597	35,850	43,409	965
0.8	19,762	26,270	33,144	-16,865
0.9	565	7,999	15,851	-55,577
1	-52,141	-42,988	-33,256	-124,238
CRRA = 5	VARIABLE ANNUITY			
VA share	Base case	-50 pts	-100 pts	Immediate annuity
0	0	0	0	0
0.1	21,237	22,407	23,949	19,872
0.2	40,132	43,103	46,268	33,668
0.3	55,593	59,644	63,927	46,230
0.4	67,656	72,399	77,387	53,162
0.5	74,995	80,438	86,139	55,791
0.6	72,368	78,300	84,519	45,119
0.7	61,392	68,034	75,037	19,373
0.8	28,456	34,106	42,769	-25,549
0.9	-26,590	-19,332	-11,633	-106,652
1	-136,355	-124,099	-111,256	-240,108

TABLE 6. *Impact of VA fees on annuity equivalent wealth, female with bequest motive*

Source: Authors' calculations. See notes to Table 5.

GLWB EXERCISE AGE						
VA share	65	70	75	80	Optimal	Immediate annuity
A: WITHOUT TAX CONSIDERATION						
0	0	0	0	0	0	0
0.1	46,481	42,967	32,393	15,132	46,481	45,141
0.2	91,144	78,022	52,480	18,098	91,144	85,229
0.3	121,481	98,011	53,276	622	121,481	108,678
0.4	149,554	113,915	47,992	-21,030	149,554	129,375
0.5	166,297	116,725	30,028	-49,714	166,297	139,952
0.6	173,045	102,895	3,220	-80,761	173,045	145,458
0.7	180,725	73,900	-22,248	-110,889	180,725	135,447
0.8	170,758	48,737	-40,948	-131,860	170,758	123,683
0.9	152,905	36,167	-60,547	-155,000	152,905	78,668
1	106,389	28,876	-70,689	-166,818	106,389	-1,707
B: WITH TAX CONSIDERATION						
0	0	0	0	0	0	0
0.1	44,758	41,236	30,709	14,014	44,758	42,275
0.2	84,537	71,606	48,080	14,995	84,537	75,468
0.3	111,446	88,188	46,501	-4,403	111,446	94,193
0.4	136,222	100,488	38,329	-27,393	136,222	111,113
0.5	150,127	99,618	18,096	-56,707	150,127	118,469
0.6	154,091	81,916	-9,541	-87,424	154,091	120,673
0.7	158,756	53,613	-34,751	-116,703	158,756	108,360
0.8	144,927	31,243	-52,180	-136,935	144,927	92,625
0.9	122,850	19,718	-70,224	-158,936	122,850	49,961
1	79,345	13,154	-79,165	-169,429	79,345	-31,342

TABLE 7A. *Impact of taxation on annuity equivalent wealth (dollars), $1 million financial assets, female with bequest motive, CRRA = 2*

Source: Authors' calculations. Household is assumed to have $26,000 Social Security income and $1 million financial assets. CRRA refers to the coefficient of relative risk aversion, with 5 being more risk averse than 2. VA share refers to the share of financial assets invested in a variable annuity. Annuity equivalent wealth is the factor by which the wealth of someone aged 65 who is unable to purchase an annuity must be increased to make them just as well off (in expected utility terms) as the same individual with access to the annuity market. The bequest motive is parameterized as in Lockwood (2012). An inflation rate of 2% is assumed.

GLWB EXERCISE AGE						
VA share	65	70	75	80	Optimal	Immediate annuity
A: WITHOUT TAX CONSIDERATION						
0	0	0	0	0	0	0
0.1	152,293	159,944	149,253	113,045	161,393	151,939
0.2	270,828	270,443	242,533	176,861	273,436	274,101
0.3	331,274	320,373	271,230	172,290	331,274	333,769
0.4	378,688	355,939	281,344	154,040	378,688	380,984
0.5	425,587	370,744	264,496	111,140	425,587	433,175
0.6	453,503	361,024	221,725	54,199	453,503	479,814
0.7	473,743	328,876	169,367	11,737	473,743	476,196
0.8	461,395	275,050	119,904	-25,891	461,395	477,438
0.9	418,182	239,982	63,253	-76,595	418,182	385,448
1	322,238	219,337	42,858	-110,558	322,238	235,739
B: WITH TAX CONSIDERATION						
0	0	0	0	0	0	0
0.1	143,818	150,830	140,011	105,258	152,225	144,924
0.2	257,925	256,948	228,460	163,345	259,881	262,026
0.3	317,570	306,163	254,938	157,320	317,570	321,579
0.4	365,196	341,145	263,952	138,404	365,196	369,412
0.5	398,495	355,476	247,646	96,703	398,495	410,352
0.6	425,544	345,332	209,249	48,377	425,544	455,950
0.7	446,630	315,548	161,854	7,038	446,630	455,216
0.8	435,051	275,745	115,440	-29,194	435,051	460,226
0.9	399,520	245,233	64,407	-73,366	399,520	388,643
1	335,423	227,543	46,286	-102,999	335,423	271,776

TABLE 7B. *Impact of taxation on annuity equivalent wealth (dollars), $1 million financial assets, female with strong bequest motive, CRRA = 5*

Source: Authors' calculations. Notes: See Table 7A

CRRA = 2	GLWB EXERCISE AGE					
VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	9,055	7,133	3,795	-471	9,055	7,200
0.2	17,006	12,349	4,975	-4,234	17,006	12,439
0.3	21,230	14,009	1,749	-13,110	21,230	14,959
0.4	25,264	13,507	-5,823	-26,569	25,264	15,787
0.5	27,052	10,136	-17,793	-42,866	27,052	15,232
0.6	26,139	-327	-38,832	-62,725	26,139	10,858
0.7	23,150	-17,176	-62,840	-76,499	23,150	965
0.8	12,517	-46,356	-76,227	-84,738	12,517	-16,865
0.9	-6,958	-69,876	-81,715	-90,925	-6,958	-55,577
1	-61,367	-72,114	-84,768	-94,457	-61,367	-124,238
CRRA = 5	GLWB EXERCISE AGE					
VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	20,923	20,318	17,329	12,325	20,945	19,872
0.2	39,456	35,096	27,053	15,543	39,456	33,668
0.3	54,908	46,989	28,041	8,255	54,908	46,230
0.4	67,325	50,889	19,509	-9,452	67,325	53,162
0.5	75,212	46,617	2,171	-34,464	75,212	55,791
0.6	73,492	24,029	-35,461	-70,290	73,492	45,119
0.7	63,989	-8,642	-81,232	-96,186	63,989	19,373
0.8	32,204	-69,778	-122,484	-113,324	32,204	-25,549
0.9	-20,632	-142,146	-137,869	-123,352	-20,632	-106,652
1	-153,023	-149,201	-142,594	-129,858	-124,552	-240,108

TABLE 8. *Annuity equivalent wealth for RILA buffer, female with strong bequest motive*

Source: Authors' calculations. Household is assumed to have $26,000 Social Security income and $400,000 financial assets. [-10%,15.8%] nominal floor and ceiling, 100% participation, and 5.7% expected stock return. CRRA refers to the coefficient of relative risk aversion, with 5 being more risk averse than 2. VA share refers to the share of financial assets invested in a variable annuity. Annuity equivalent wealth is the factor by which the wealth of someone aged 65 who is unable to purchase an annuity must be increased to make them just as well off (in expected utility terms) as the same individual with access to the annuity market. The bequest motive is parameterized as in Lockwood (2012). An inflation rate of 2% is assumed.

CRRA = 2	GLWB EXERCISE AGE					
VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	6,432	3,059	-1,626	-6,976	6,432	7,200
0.2	12,049	4,624	-5,261	-16,427	12,049	12,439
0.3	13,832	2,635	-13,086	-30,609	13,832	14,960
0.4	15,871	-1,002	-24,568	-48,852	15,871	15,787
0.5	15,471	-7,561	-40,242	-69,332	15,471	15,232
0.6	12,914	-20,704	-64,767	-90,530	12,914	10,858
0.7	8,216	-40,279	-88,332	-108,860	8,216	965
0.8	-4,936	-71,277	-105,603	-119,627	-4,936	-16,865
0.9	-26,990	-94,344	-111,638	-126,194	-26,990	-55,577
1	-81,943	-96,236	-114,404	-129,398	-81,943	-124,238
CRRA = 5	GLWB EXERCISE AGE					
VA share	65	70	75	80	Optimal	Immediate annuity
0	0	0	0	0	0	0
0.1	17,894	15,529	10,788	4,259	17,894	19,872
0.2	31,981	26,058	15,734	1,793	31,981	33,668
0.3	43,466	30,499	12,717	-10,209	43,466	46,230
0.4	53,648	30,663	s1,166	-31,852	53,648	53,163
0.5	59,052	25,478	-18,904	-61,443	59,052	55,791
0.6	55,735	5,469	-59,492	-93,789	55,735	45,119
0.7	43,993	-30,054	-100,986	-127,428	43,993	19,373
0.8	16,267	-87,378	-148,002	-149,020	16,267	-25,550
0.9	-41,390	-159,552	-162,791	-157,763	-41,390	-106,652
1	-170,561	-169,140	-166,601	-162,491	-155,230	-240,108

TABLE 9. *Annuity equivalent wealth for RILA fixed, female with strong bequest motive*

Source: Authors' calculations. Notes: See Table 8.